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BASE ISOLATION FROM SEISMIC WAVES BY A VISCOELASTIC LAYER

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In the present paper we study harmonic oscillations of elastic rectangle above a viscoelastic layered half-space. The latter consists of an elastic half-space to which a viscoelastic layer is embedded at a certain depth. By combining Fourier integral transform in the half-space and series representation of the solution in the rectangle the problem is reduced to an integral equation over the base of the rectangle. By solving this integral equation we investigate the possibility of base isolation in dependence upon viscoelastic properties of the intermediate layer as well as upon geometrical and physical parameters of the materials.

1. Introduction

The importance of the problem about base isolation of constructions in seismic zones is very important both from practical and theoretical points of view. Various approaches are applied to this problem, and one of the most efficient method in the engineering practice is protection of the constructions by some damping materials embedded in the soil foundation. A good survey to the problem is given in [1].

The importance of the problem of base isolation can be understood following the survey of such works as [1–3]. Some interesting results can be found in [4, 5]. In the paper of de la Cruz, Hube, and Spanos [5] the authors continue to develop the geophysics model of porous elastic media previously studied in their work [6]. They study the mode conversion and proportion between the energies in the reflected and transmitted waves. In [7] the authors study the reflection of seismic waves from the free boundary of porous foundation whose mechanical properties are described by the Goodman–Cowin–Nunziato model.

Analogous investigations were carried out by some authors for viscoelastic materials. For example, in [8] there is considered an inhomogeneous viscoelastic

[1]Sumbatyan Mezhlum (sumbat@math.rsu.ru), Faculty of Mechanics and Mathematics, Rostov State University, Russia 344090, Rostov-on-Don, Milchakova Street, 8a.

[2]Zampoli Vittorio, Vaccaro Massimo, D.I.I.M.A., University of Salerno, Italy 84084 Fisciano (Salerno), (vzampoli@unisa.it), (massimo_vaccaro@libero.it)

layered isotropic medium, and the authors study wave propagation, reflection and transmission in such media. Attenuation and phase shifts are also the subjects of the study. The obtained results are valid in a wide seismic frequency band being compared with predictions by some simpler and rougher methods.

It should be noted that some classical monographs are devoted to the general theory of dynamic properties of viscoelastic materials, as well as to interaction of such materials with seismic waves. We can refer to the book of D.I.G. Jones [9], where the author gives the survey of various approaches to practical structural applications. More theoretical methods to the problem in concern are applied in [10]. For transient problems there are applied the Laplace integral transform as well as time-domain calculations. A chapter devoted to damping of vibrations by viscoelastic materials can be found in [11]. Finally, general background to the problem of seismic isolation is presented in [12]. Some interesting and important results, as well as further helpful references can be found in [13, 14] and some other articles cited there.

In this work, we study the *in-plane* problem connected with the presence of a viscoelastic layer of Kelvin–Voigt type in an elastic half-space. The model is completed supposing the existence of an elastic solid placed on the free boundary of the half-space. In particular, we investigate the displacement vector characterizing behaviour of the elastic solid, in its dependence on several parameters.

2. Formulation of the Problem

Let us consider the two-dimensional plane-strain problem about the incidence of a plane wave in an viscoelastic half-space (see Fig. 1). The half-space consists of a homogeneous linear isotropic elastic material, where a different viscoelastic layer of the thickness h_1 is placed on the depth h.

The plane-strain formulation implies the displacement vector $\mathbf{u}$ to be of the form:

$$\mathbf{u}(x, y, z, t) = \{u_x(x, y, t), u_y(x, y, t), 0\}\,. \tag{2.1}$$

In frames of this theory the equation of motion of the elastic medium [15]

$$\mu \ \Delta \ \mathbf{u} + (\lambda + \mu) \ \text{grad} \ \text{div} \ \mathbf{u} = \rho \frac{\partial^2 \mathbf{u}}{\partial t^2}\,, \tag{2.2}$$

can be represented as a combination of two elastic potentials: φ and ψ that is known in literature as the Lame representation [15]:

$$\begin{gathered} u_x = u_1 = \frac{\partial \varphi}{\partial x} + \frac{\partial \psi}{\partial y}\,, \qquad u_y = u_2 = \frac{\partial \varphi}{\partial y} - \frac{\partial \psi}{\partial x}\,, \\ \varphi(x, y, t) = e^{-i\omega t}\varphi(x, y)\,, \qquad \psi(x, y, t) = e^{-i\omega t}\psi(x, y)\,, \end{gathered} \tag{2.3}$$

if the process is assumed to be harmonic with respect to time, with the angular frequency ω. Then Eq. (2.2) degenerates to a couple of the (wave) Helmhotlz equations

$$\Delta\varphi + k_p^2\varphi = 0\,, \qquad \Delta\psi + k_s^2\psi = 0\,, \qquad (k_p = \omega/c_p\,, \quad k_s = \omega/c_s)\,, \tag{2.4}$$

where $c_p^2 = (\lambda+2\mu)/\rho$ and $c_s^2 = \mu/\rho$ determine the longitudinal and transverse wave speeds, respectively.

With all that the only nontrivial components of the stress tensor are

$$\sigma_{xx} = \sigma_{11} = (\lambda + 2\mu)\frac{\partial u_1}{\partial x} + \lambda\frac{\partial u_2}{\partial y}\,, \quad \sigma_{xy} = \sigma_{12} = \mu\left(\frac{\partial u_1}{\partial y} + \frac{\partial u_2}{\partial x}\right)\,,$$
$$\sigma_{yy} = \sigma_{22} = (\lambda + 2\mu)\frac{\partial u_2}{\partial y} + \lambda\frac{\partial u_1}{\partial x}\,. \tag{2.5}$$

Let us assume that above the considered elastic half-space there is placed an elastic rectangle joined with the half-space. Then the governing equations for the rectangle have an analogous form like in Eqs. (2.1)–(2.5). There arise some other physical quantities which we mark by the sub- or superscript "2".

Finally, we assume that a viscoelastic layer is placed inside the main elastic half-space, with the constitutive equation being given by the simplest Kelvin–Voight model. It is known [9] that in the harmonic regime the governing equations of such a viscoelastic media have the same form (2.1)–(2.5) as in the elastic case, but with complex-valued physical moduli instead of real ones. We denote the elastic moduli for the viscoelastic layer as λ_1^* and μ_1^*, namely $\lambda_1^* = \lambda_1 - i\eta_{\lambda 1}$, and $\mu_1^* = \mu_1 - i\eta_{\mu 1}$. It is clear, both physically and mathematically, that the wave process in the viscoelastic layer is damping with distance, with respective complex-valued wave numbers k_p^* and k_s^*. All physical quantities in the viscoelastic layer are marked by the sub- or super- script "1". The upper elastic layer will be marked by the number "0".

Let us assume that a plane seismic harmonic wave is incident from below. If the wave is longitudinal and the angle of incidence with respect to the vertical axis is θ then

$$u_1^{inc} = \sin\theta\, e^{ik_p[x\sin\theta + (y+h_0)\cos\theta]}\,,$$
$$u_2^{inc} = \cos\theta\, e^{ik_p[x\sin\theta + (y+h_0)\cos\theta]}\,, \tag{2.6}$$
$$\mathbf{u}(x,y) = \mathbf{u}^{inc}(x,y) + \mathbf{u}^{sc}(x,y)\,, \qquad (h_0 = h + h_1)\,.$$

The posed problem simulates the protection of the civil engineering structures from the incident seismic waves by absorbing layers.

3. Solution in the Half-Plane

Let us apply the Fourier transform along the horizontal coordinate x, which is defined for arbitrary function $f(x,y)$ as

$$F(s,y) = \int_{-\infty}^{\infty} f(x,y)e^{isx}dx\,, \qquad f(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(s,y)e^{-isx}ds\,. \tag{3.1}$$

Then equations of motion (2.4) are reduced to the ordinary differential equations with constant coefficients

$$\tilde{\varphi}''(s,y) - \left(s^2 - k_p^2\right)\tilde{\varphi}(s,y) = 0, \qquad \tilde{\psi}''(s,y) - \left(s^2 - k_s^2\right)\tilde{\psi}(s,y) = 0, \tag{3.2}$$

where primes designate the derivative with respect to y. Note that these equations are valid for all domains, with respective physical parameters. Note that tilde above any physical quantity means its Fourier transform.

The solution to Eqs. (3.2) is given as follows.

1. Elastic half-space ($y < -h_0$):

$$\tilde{\varphi}^{sc}(s,y) = A_1(s)\, e^{\gamma_p(s)(y+h_0)}, \qquad \gamma_p(s) = \sqrt{s^2 - k_p^2}, \tag{3.3a}$$

$$\tilde{\psi}^{sc}(s,y) = A_2(s)\, e^{\gamma_s(s)(y+h_0)}, \qquad \gamma_s(s) = \sqrt{s^2 - k_s^2}, \tag{3.3b}$$

since the diffracted wave field in the lowest domain satisfies the Helmholtz equation too. When constructing these solutions, we have taken into account that the scattered wave field should satisfy the radiation condition as $y \to -\infty$ [15].

2. Viscoelastic layer ($h_0 = h + h_1$, $-h_0 \leqslant y \leqslant -h.$):

$$\tilde{\varphi}^{(1)}(s,y) = B_1(s)\,\mathrm{ch}\left[\gamma_p^*(s)(y+h)\right] + B_2(s)\,\mathrm{sh}\left[\gamma_p^*(s)(y+h)\right], \tag{3.4a}$$

$$\tilde{\psi}^{(1)}(s,y) = C_1(s)\,\mathrm{ch}\left[\gamma_s^*(s)(y+h)\right] + C_2(s)\,\mathrm{sh}\left[\gamma_s^*(s)(y+h)\right], \tag{3.4b}$$

$$\gamma_p^*(s) = \sqrt{s^2 - k_p^{*2}}\ , \quad \gamma_s^*(s) = \sqrt{s^2 - k_s^{*2}}, \quad \mathrm{sh} = \sinh, \quad \mathrm{ch} = \cosh. \tag{3.4c}$$

3. The upper elastic layer ($-h \leqslant y \leqslant 0$):

$$\tilde{\varphi}^{(0)}(s,y) = D_1(s)\,\mathrm{ch}\left[\gamma_p(s)\ \ y\right] + D_2(s)\,\mathrm{sh}\left[\gamma_p(s)\ \ y\right], \tag{3.5a}$$

$$\tilde{\psi}^{(0)}(s,y) = E_1(s)\,\mathrm{ch}\left[\gamma_s(s)\ \ y\right] + E_2(s)\,\mathrm{sh}\left[\gamma_s(s)\ \ y\right]. \tag{3.5b}$$

It should be noted that the unknown quantities $A_1(s)$, $A_2(s)$, $B_1(s)$, $B_2(s)$, $C_1(s)$, $C_2(s)$, $D_1(s)$, $D_2(s)$, $E_1(s)$, $E_2(s)$ can be found by satisfying the boundary conditions, which in our problem are conditions of continuity of the displacement and stress fields over all interface boundaries:

Conditions of continuity at $y = -h_0$:

$$u_1^{inc}(s,\ \ y) + u_1^{sc}(s,\ \ y) = u_1^{(1)}(s,\ \ y), \tag{3.6a}$$

$$u_2^{inc}(s,\ \ y) + u_2^{sc}(s,\ \ y) = u_1^{(1)}(s,\ \ y), \tag{3.6b}$$

$$\sigma_{22}^{inc}(s,\ \ y) + \sigma_{22}^{sc}(s,\ \ y) = \sigma_{22}^{(1)}(s,\ \ y), \tag{3.6c}$$

$$\sigma_{12}^{inc}(s,\ \ y) + \sigma_{12}^{sc}(s,\ \ y) = \sigma_{12}^{(1)}(s,\ \ y). \tag{3.6d}$$

Conditions of continuity at $y = -h$:

$$u_1^{(1)}(s,\ \ y) = u_1^{(0)}(s,\ \ y), \tag{3.7a}$$

$$u_2^{(1)}(s, \ y) = u_2^{(0)}(s, \ y)\,, \tag{3.7b}$$

$$\sigma_{22}^{(1)}(s, \ y) = \sigma_{22}^{(0)}(s, \ y)\,, \tag{3.7c}$$

$$\sigma_{12}^{(1)}(s, \ y) = \sigma_{12}^{(0)}(s, \ y)\,. \tag{3.7d}$$

If we assume for a while that the complex-valued amplitude of the surface normal and shear stresses are certain functions $\sigma(x)$ and $\tau(x)$, then

Conditions on the upper boundary surface at $y = 0$:

$$\sigma_{22}^{(0)}(s, \ y) = \sigma(s)\,, \tag{3.8a}$$

$$\tau_{12}^{(0)}(s, \ y) = \tau(s)\,. \tag{3.8b}$$

It is obvious that the ten unknown quantities $A_1(s) - E_2(s)$ in can be determined from the linear algebraic system of ten equations (3.6)–(3.8).

In order to satisfy conditions (3.6)–(3.8), let us write out respective physical fields in all three domains.

The displacements and stresses in the lower elastic half-plane:

$$u_1^{sc}(s, y) = A_1(is)\, e^{\gamma_p(y+h_0)} + A_2\gamma_s e^{\gamma_s(y+h_0)}, \tag{3.9a}$$

$$u_2^{sc}(s, y) = A_1\gamma_p e^{\gamma_p(y+h_0)} - A_2(is)\, e^{\gamma_s(y+h_0)}, \tag{3.9b}$$

$$\begin{aligned}\sigma_{22}^{sc}(s, y) = (\lambda + 2\mu)\Big\{A_1\gamma_p^2 e^{\gamma_p(y+h_0)} - A_2(is)\,\gamma_s e^{\gamma_s(y+h_0)} + \\ +\lambda(is)\Big[A_1(is)\, e^{\gamma_p(y+h_0)} + A_2\gamma_s e^{\gamma_s(y+h_0)}\Big]\Big\},\end{aligned} \tag{3.9c}$$

$$\begin{aligned}\sigma_{12}^{sc}(s, y) = \mu\Big[A_1(is)\,\gamma_p e^{\gamma_p(y+h_0)} - A_2\gamma_s^2 e^{\gamma_s(y+h_0)} + \\ +A_1(is)\,\gamma_p e^{\gamma_p(y+h_0)} + A_2(is)^2\, e^{\gamma_s(y+h_0)}\Big].\end{aligned} \tag{3.9d}$$

The displacements and stresses in the viscoelastic layer:

$$\begin{aligned}u_1^{(1)}(s, y) = (is)\Big\{B_1\mathrm{ch}\Big[\gamma_p^*(y+h)\Big] + B_2\mathrm{sh}\Big[\gamma_p^*(y+h)\Big]\Big\} \\ + \{C_1\gamma_s^*\mathrm{sh}[\gamma_s^*(y+h)] + C_2\gamma_s^*\mathrm{ch}[\gamma_s^*(y+h)]\}\,,\end{aligned} \tag{3.10a}$$

$$\begin{aligned}u_2^{(1)}(s, y) = \Big\{B_1\gamma_p^*\mathrm{sh}\Big[\gamma_p^*(y+h)\Big] + B_2\gamma_p^*\mathrm{ch}\Big[\gamma_p^*(y+h)\Big]\Big\} - \\ -(is)\{C_1\mathrm{ch}[\gamma_s^*(y+h)] + C_2\mathrm{sh}[\gamma_s^*(y+h)]\}\,,\end{aligned} \tag{3.10b}$$

$$\begin{aligned}\sigma_{22}^{(1)}(s, y) = (\lambda^* + 2\mu^*)\Big\langle[\gamma_s^*]^2\Big\{B_1\mathrm{ch}\Big[\gamma_p^*(y+h)\Big] + B_2\mathrm{sh}\Big[\gamma_p^*(y+h)\Big]\Big\} - \\ -(is)\,\gamma_s^*\{C_1\mathrm{sh}[\gamma_s^*(y+h)] + C_2\mathrm{ch}[\gamma_s^*(y+h)]\}\rangle + \\ +\lambda^*(is)\Big\langle(is)\Big\{B_1\mathrm{ch}\Big[\gamma_p^*(y+h)\Big] + B_2\mathrm{sh}\Big[\gamma_p^*(y+h)\Big]\Big\} + \\ +\gamma_s^*\{C_1\mathrm{sh}[\gamma_s^*(y+h)] + C_2\mathrm{ch}[\gamma_s^*(y+h)]\}\rangle\,,\end{aligned} \tag{3.10c}$$

$$\sigma_{12}^{(1)}(s,y) = \mu^* \Big\langle (is)\,\gamma_p^* \Big\{ B_1 \mathrm{sh}\big[\gamma_p^*(y+h)\big] + B_2 \mathrm{ch}\big[\gamma_p^*(y+h)\big]\Big\} + \\ + [\gamma_s^*]^2 \{C_1 \mathrm{ch}[\gamma_s^*(y+h)] + C_2 \mathrm{sh}[\gamma_s^*(y+h)]\} + \\ + (is)\Big[\gamma_p^* \Big\{ B_1 \mathrm{sh}\big[\gamma_p^*(y+h)\big] + B_2 \mathrm{ch}\big[\gamma_p^*(y+h)\big]\Big\} - \\ - (is)\{C_1 \mathrm{ch}[\gamma_s^*(y+h)] + C_2 \mathrm{sh}[\gamma_s^*(y+h)]\}\Big]\Big\rangle . \tag{3.10d}$$

The displacements and stresses in the upper elastic layer

$$u_1^{(0)}(s,y) = (is)\Big[D_1 \mathrm{ch}\big(\gamma_p\ y\big) + D_2 \mathrm{sh}\big(\gamma_p\ y\big)\Big] + \\ + \gamma_s [E_1 \mathrm{sh}(\gamma_s\ y) + E_2 \mathrm{ch}(\gamma_s\ y)], \tag{3.11a}$$

$$u_2^{(0)}(s,y) = \gamma_p \Big[D_1 \mathrm{sh}\big(\gamma_p\ y\big) + D_2 \mathrm{ch}\big(\gamma_p\ y\big)\Big] - \\ - (is)[E_1 \mathrm{ch}(\gamma_s\ y) + E_2 \mathrm{sh}(\gamma_s\ y)], \tag{3.11b}$$

$$\sigma_{22}^{(0)}(s,y) = (\lambda + 2\mu)\Big\{\gamma_p^2 \Big[D_1 \mathrm{ch}\big(\gamma_p\ y\big) + D_2 \mathrm{sh}\big(\gamma_p\ y\big)\Big] - \\ - (is)\,\gamma_s [E_1 \mathrm{sh}(\gamma_s\ y) + E_2 \mathrm{ch}(\gamma_s\ y)]\} + \\ + \lambda (is)\Big\{(is)\Big[D_1 \mathrm{ch}\big(\gamma_p\ y\big) + D_2 \mathrm{sh}\big(\gamma_p\ y\big)\Big] + \\ + \gamma_s [E_1 \mathrm{sh}(\gamma_s\ y) + E_2 \mathrm{ch}(\gamma_s\ y)]\}, \tag{3.11c}$$

$$\sigma_{12}^{(0)}(s,y) = \mu \Big\langle (is)\,\gamma_p \Big[D_1 \mathrm{sh}\big(\gamma_p\ y\big) + D_2 \mathrm{ch}\big(\gamma_p\ y\big)\Big] + \\ + \gamma_s^2 [E_1 \mathrm{ch}(\gamma_s\ y) + E_2 \mathrm{sh}(\gamma_s\ y)] + \\ + (is)\Big\{\gamma_p \Big[D_1 \mathrm{sh}\big(\gamma_p\ y\big) + D_2 \mathrm{ch}\big(\gamma_p\ y\big)\Big] - \\ - (is)[E_1 \mathrm{ch}(\gamma_s\ y) + E_2 \mathrm{sh}(\gamma_s\ y)]\}\Big\rangle . \tag{3.11d}$$

Then conditions (3.6)–(3.8) result in the following 10×10 system of linear algebraic equations for the unknown coefficients:

$$y = -h_0 :$$

$$(is)\ \ A_1 + \gamma_s\ \ A_2 - (is)\,\mathrm{ch}\big(\gamma_p^*\ \ h_1\big) B_1 + (is)\,\mathrm{sh}\big(\gamma_p^*\ \ h_1\big) B_2 + \\ + \gamma_s^* \mathrm{sh}(\gamma_s^*\ \ h_1)\, C_1 - \gamma_s^* \mathrm{ch}(\gamma_s^*\ \ h_1)\, C_2 = -2\pi \sin\theta\ \ \delta\big(s - k_p \sin\theta\big), \tag{3.12a}$$

$$\gamma_p\ \ A_1 - (is)\ \ A_2 - \gamma_p^* \mathrm{sh}\big(\gamma_p^*\ \ h_1\big) B_1 + \gamma_p^* \mathrm{ch}\big(\gamma_p^*\ \ h_1\big) B_2 + \\ + (is)\,\mathrm{ch}(\gamma_s^*\ \ h_1)\, C_1 - (is)\,\mathrm{sh}(\gamma_s^*\ \ h_1)\, C_2 = -2\pi \cos\theta\delta\big(s - k_p \sin\theta\big), \tag{3.12b}$$

$$\Big[(\lambda + 2\mu)\,\gamma_p^2 - \lambda s^2\Big] A_1 - (is)\,\gamma_s 2\mu A_2 - \\ - \Big[(\lambda^* + 2\mu^*)\,\gamma_p^2 - \lambda^* s^2\Big]\,\mathrm{ch}\big(\gamma_p^* h_1\big) B_1 + \\ + \Big[(\lambda^* + 2\mu^*)\,\gamma_p^2 - \lambda^* s^2\Big]\,\mathrm{sh}\big(\gamma_p^*\ \ h_1\big) B_1 - \\ - (is)\,\gamma_s^*\ \ 2\mu^* \mathrm{sh}(\gamma_s^*\ \ h_1)\, C_1 + (is)\,\gamma_s^*\ \ 2\mu^* \mathrm{ch}(\gamma_s^*\ \ h_1)\, C_2 = \tilde{\sigma}_{22}^{inc}, \tag{3.12c}$$

$$\mu\left[2(is)\ \ \gamma_p\ \ A_1+\left(\gamma_s^2+s^2\right)\ \ A_2\right]\ -$$
$$-\mu^*\left\langle -2(is)\ \ \gamma_p^*\mathrm{sh}\left(\gamma_p^*\ \ h_1\right)B_1+2(is)\ \ \gamma_p^*\mathrm{ch}\left(\gamma_p^*\ \ h_1\right)B_2+\right. \tag{3.12d}$$
$$\left.+\left(\gamma_s^{*2}+s^2\right)\mathrm{ch}\left(\gamma_s^*\ \ h_1\right)C_1-\left(\gamma_s^{*2}+s^2\right)\mathrm{sh}\left(\gamma_s^*\ \ h_1\right)C_2\right\rangle=\tilde{\sigma}_{12}^{inc},$$

$$y=-h:$$

$$(is)\ \ B_1+\gamma_s^*\ \ C_2-(is)\,\mathrm{ch}\left(\gamma_p\ \ h\right)D_1+$$
$$+(is)\,\mathrm{sh}\left(\gamma_p\ \ h\right)D_2+\gamma_s\mathrm{sh}\left(\gamma_s\ \ h\right)E_1-\gamma_s\mathrm{ch}\left(\gamma_s\ \ h\right)E_2=0, \tag{3.13a}$$

$$\gamma_p^*\ \ B_2-(is)\ \ C_1-\gamma_p\mathrm{sh}\left(\gamma_p\ \ h\right)D_1-$$
$$-\gamma_p\mathrm{ch}\left(\gamma_p\ \ h\right)D_2+(is)\,\mathrm{ch}\left(\gamma_s\ \ h\right)E_1-(is)\,\mathrm{sh}\left(\gamma_s\ \ h\right)E_2=0, \tag{3.13b}$$

$$\left[(\lambda^*+2\mu^*)\,\gamma_p^{*2}-\lambda^*s^2\right]\ \ B_1-2\mu^*(is)\ \ \gamma_s^*\ \ C_2-$$
$$-\left[(\lambda+2\mu)\,\gamma_p^2-\lambda s^2\right]\mathrm{ch}\left(\gamma_p\ \ h\right)D_1+$$
$$+\left[(\lambda+2\mu)\,\gamma_p^2-\lambda s^2\right]\mathrm{sh}\left(\gamma_p\ \ h\right)D_2- \tag{3.13c}$$
$$-2\mu(is)\ \ \gamma_s\mathrm{sh}\left(\gamma_s\ \ h\right)E_1+2\mu(is)\ \ \gamma_s\ \ \mathrm{ch}\left(\gamma_s\ \ h\right)E_2=0,$$

$$\mu^*\left[2(is)\ \ \gamma_p^*\ \ B_2+\left(\gamma_s^{*2}+s^2\right)\ \ C_1\ \ \right]-$$
$$-\mu\left\{-2(is)\ \ \gamma_p\mathrm{sh}\left(\gamma_p\ \ h\right)D_1+2(is)\ \ \gamma_p\mathrm{ch}\left(\gamma_p\ \ h\right)D_2+\right. \tag{3.13c}$$
$$\left.+\left(\gamma_s^2+s^2\right)\mathrm{ch}\left(\gamma_p\ \ h\right)E_1-\left(\gamma_s^2+s^2\right)\mathrm{sh}\left(\gamma_p\ \ h\right)E_2\right\}=0,$$

$$y=0:$$

$$\left[(\lambda+2\mu)\ \ \gamma_p^2-\lambda s^2\right]D_1-2\mu(is)\ \ \gamma_sE_2=\tilde{\sigma}(s), \tag{3.14a}$$

$$\mu\left[2(is)\ \ \gamma_sD_2+\left(\gamma_s^2+s^2\right)E_1\right]=\tilde{\tau}(s), \tag{3.14b}$$

where $\tilde{\sigma}(s)$ and $\tilde{\tau}(s)$ are the Fourier transforms (images) of the normal and tangential stresses at $y=0$, which for a while are accepted to be known.

This 10×10 linear algebraic system (3.12)–(3.14) has the following form

$$\begin{pmatrix}
a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} & 0 & 0 & 0 & 0\\
a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} & 0 & 0 & 0 & 0\\
a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} & 0 & 0 & 0 & 0\\
a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} & 0 & 0 & 0 & 0\\
0 & 0 & a_{53} & 0 & 0 & a_{56} & a_{57} & a_{58} & a_{59} & a_{5,10}\\
0 & 0 & 0 & a_{64} & a_{65} & 0 & a_{67} & a_{68} & a_{69} & a_{6,10}\\
0 & 0 & a_{73} & 0 & 0 & a_{76} & a_{77} & a_{78} & a_{79} & a_{7,10}\\
0 & 0 & 0 & a_{84} & a_{85} & 0 & a_{87} & a_{88} & a_{89} & a_{8,10}\\
0 & 0 & 0 & 0 & 0 & 0 & a_{97} & 0 & 0 & a_{9,10}\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{10,8} & a_{10,9} & 0
\end{pmatrix}
\begin{pmatrix} A_1\\ A_2\\ B_1\\ B_2\\ C_1\\ C_2\\ D_1\\ D_2\\ E_1\\ E_2\end{pmatrix}
=
\begin{pmatrix} b_1\\ b_2\\ b_3\\ b_4\\ 0\\ 0\\ 0\\ 0\\ \tilde{\sigma}\\ \tilde{\tau}\end{pmatrix} \tag{3.15}$$

with evident representations both for the matrix elements and for elements of the right-hand side.

The solution to system (3.15) can be constructed with the use of Cramer's rule which results in the following expressions:

$$D_1 = \frac{b_1 A_{17} + b_2 A_{27} + b_3 A_{37} + b_4 A_{47} + \tilde{\sigma}(s) A_{97} + \tilde{\tau}(s) A_{10,7}}{\Delta}, \tag{3.16a}$$

$$D_2 = \frac{b_1 A_{18} + b_2 A_{28} + b_3 A_{38} + b_4 A_{48} + \tilde{\sigma}(s) A_{98} + \tilde{\tau}(s) A_{10,8}}{\Delta}, \tag{3.16b}$$

$$E_1 = \frac{b_1 A_{19} + b_2 A_{29} + b_3 A_{39} + b_4 A_{49} + \tilde{\sigma}(s) A_{99} + \tilde{\tau}(s) A_{10,9}}{\Delta}, \tag{3.16c}$$

$$E_2 = \frac{b_1 A_{1,10} + b_2 A_{2,10} + b_3 A_{3,10} + b_4 A_{4,10} + \tilde{\sigma}(s) A_{9,10} + \tilde{\tau}(s) A_{10,10}}{\Delta}, \tag{3.16d}$$

where Δ is the principal determinant of the system. Other six unknown coefficients have a form very similar to (3.16).

So far as the system (3.15) is resolved, the components of the displacement vector in the upper elastic layer can be written as follows

$$\begin{aligned} \tilde{u}_1^{(0)}(s,y) = (is)\left\{D_1(s)\cosh\left[\gamma_p(s)\ \ y\right] + D_2(s)\sinh\left[\gamma_p(s)\ \ y\right]\right\} + \\ +\gamma_s^{(s)}\left\{E_1(s)\sinh\left[\gamma_s(s)\ \ y\right] + E_2(s)\cosh\left[\gamma_s(s)\ \ y\right]\right\}, \end{aligned} \tag{3.17a}$$

$$\begin{aligned} \tilde{u}_2^{(0)}(s,y) = \gamma_p^{(s)}\left\{D_1(s)\sinh\left[\gamma_p(s)\ \ y\right] + D_2(s)\cosh\left[\gamma_p(s)\ \ y\right]\right\} - \\ -(is)\left\{E_1(s)\cosh\left[\gamma_s(s)\ \ y\right] + E_2(s)\sinh\left[\gamma_s(s)\ \ y\right]\right\}. \end{aligned} \tag{3.17b}$$

This results in the following expressions on the upper boundary surface, which are the most important quantities when arranging the coupling conditions between the vibrating foundation and the oscillating elastic rectangular solid:

$$\begin{aligned} \tilde{u}_1^{(0)}(s,0) &= (is)\ \ D_1(s) + \gamma_s(s)\ \ E_2(s), \\ \tilde{u}_2^{(0)}(s,0) &= \gamma_p(s)\ \ D_2(s) - (is)\ \ E_1(s). \end{aligned} \tag{3.18}$$

By substituting (3.16) into (3.18) the last expressions can be rewritten in the more concrete form

$$\begin{aligned} \tilde{u}_1^{(0)}(s,0) &= f_1^{\sigma}(s)\,\tilde{\sigma}(s) + f_1^{\tau}(s)\,\tilde{\tau}(s) + f_1^{(0)}(s), \\ \tilde{u}_2^{(0)}(s,0) &= f_2^{\sigma}(s)\,\tilde{\sigma}(s) + f_2^{\tau}(s)\,\tilde{\tau}(s) + f_2^{(0)}(s), \end{aligned} \tag{3.19}$$

where

$$\begin{aligned} f_1^{\sigma}(s) &= \left[isA_{97} + \gamma_s(s) A_{10,9}\right], \qquad f_2^{\sigma}(s) = \left[\gamma_p(s) A_{98} - isA_{99}\right], \\ f_1^{\tau}(s) &= \left[isA_{10,7} + \gamma_s(s) A_{10,10}\right], \quad f_2^{\tau}(s) = \left[\gamma_p(s) A_{10,8} - isA_{10,9}\right]. \end{aligned} \tag{3.20}$$

This allows us, by applying the inverse Fourier transform, to connect the two components of the displacement and two components of the contact stress over the base of the rectangle

$$
\begin{aligned}
u_1^{(0)}(x) &= \frac{1}{2\pi}\int_{-a}^{a}\sigma(\xi)\,d\xi\int_{-\infty}^{\infty} f_1^{\sigma}(s)\ \ e^{-is(x-\xi)}ds+\\
&+\frac{1}{2\pi}\int_{-a}^{a}\tau(\xi)\,d\xi\int_{-\infty}^{\infty} f_1^{\tau}(s)\ \ e^{-is(x-\xi)}ds+\frac{1}{2\pi}\int_{-\infty}^{\infty} f_1^{(0)}(s)\ \ e^{-isx}ds,\\
u_2^{(0)}(x) &= \frac{1}{2\pi}\int_{-a}^{a}\sigma(\xi)\,d\xi\int_{-\infty}^{\infty} f_2^{\sigma}(s)\ \ e^{-is(x-\xi)}ds+\\
&+\frac{1}{2\pi}\int_{-a}^{a}\tau(\xi)\,d\xi\int_{-\infty}^{\infty} f_2^{\tau}(s)\ \ e^{-is(x-\xi)}ds+\frac{1}{2\pi}\int_{-\infty}^{\infty} f_2^{(0)}(s)\ \ e^{-isx}ds,
\end{aligned}
\tag{3.21}
$$

where we have taken into account that in the case, when the free surface of the foundation is stress-free out of the construction base, functions $\sigma(x)$ and $\tau(x)$ are nontrivial only over the interval $x \in (-a, a)$.

4. Solution in the Elastic Rectangle

Let us pass to the equation of harmonic motion of the elastic rectangle. The boundary conditions over the boundary faces of the rectangular domain correspond to stress-free left, top, and right faces, and the contact conditions over the lower face.

The dynamic problem of linear elasticity for the rectangular domain in some cases, under special type of loading, admits exact explicit solutions in terms of Fourier trigonometric series. This is connected with a certain combination of normal stress and tangential displacement, or vice versa, of tangential stress and normal displacement. For our boundary conditions when the normal and tangential stresses are given over the full boundary the problem requires a numerical treatment. This can be attained by various methods, the method used in the present work is founded on Boundary Element Techniques.

Let $x = (x_1, x_2)$ and $y = (y_1, y_2)$ denote points in the considered two-dimensional space, and the full boundary of the rectangle be designated as l. Then it is well known (see, for example, [Kupradze]) that the following Betti integral identities are valid

$$
\begin{gathered}
u_k(x) = 2\int_l \mathbf{T}_y\left[\mathbf{U}^{(k)}(y,x)\right]\cdot\mathbf{u}(y)\,dl_y - 2\int_{l_4}\mathbf{U}^{(k)}(y,x)\cdot\mathbf{T}_y^{(4)}dl_y\,,\\
x, y \in l\ , \qquad l = l_1\cup l_2\cup l_3\cup l_4\,, \qquad k = 1, 2\,,
\end{gathered}
\tag{4.1}
$$

where l_1 is the left face of the rectangle, l_2 is its upper, l_3 its right face, respectively, and l_4 its lower base, so that the outward normal for each part of the boundary contour is

$$\begin{aligned}
l_1: &\quad n_1=-1\ ,n_2=0\ ,x=-a\ ,0\leqslant y\leqslant h,\\
l_2: &\quad n_1=0\ ,n_2=1\ ,-a\leqslant x\leqslant a,y=h,\\
l_3: &\quad n_1=1\ ,n_2=0,x=a\ ,0\leqslant y\leqslant h,\\
l_4: &\quad n_1=0\ ,n_2=-1\ ,-a\leqslant x\leqslant a\ ,y=0.
\end{aligned} \tag{4.2}$$

The two tensors arisen in Eq. (4.1) have the following form

$$\begin{aligned}
U_j^{(k)}(x,y) &= \frac{1}{4\rho\omega^2}\left\{\frac{\partial}{\partial y_k\partial y_j}\left[Y_0\left(k_p r\right)-Y_0\left(k_s r\right)\right]-\delta_{kj}k_s^2 Y_0\left(k_s r\right)\right\}=\\
&= \frac{1}{4\rho\omega^2}\left\{\frac{\partial}{\partial y_j}\left\langle\left[\frac{\partial Y_0\left(k_p r\right)}{\partial r}-\frac{\partial Y_0\left(k_s r\right)}{\partial r}\right]\frac{\partial r}{\partial y_k}\right\rangle-\delta_{kj}k_s^2 Y_0\left(k_s r\right)\right\}=\\
&= \frac{1}{4\mu k_s^2}\left\{\left[\frac{\partial^2 Y_0\left(k_p r\right)}{\partial r^2}-\frac{\partial^2 Y_0\left(k_s r\right)}{\partial r^2}\right]\frac{\partial r}{\partial y_j}\frac{\partial r}{\partial y_k}-\delta_{kj}k_s^2 Y_0\left(k_s r\right)+\right.\\
&\left.+\left[\frac{\partial Y_0\left(k_p r\right)}{\partial r}-\frac{\partial Y_0\left(k_s r\right)}{\partial r}\right]\frac{\partial^2 r}{\partial y_k\partial y_j}\right\},\quad r=|x-y|,
\end{aligned} \tag{4.3}$$

where $Y_0(x)$ is the Bessel function of the second kind and the zero's order called also Neumann or Weber function.

The partial derivative applied to tensor (4.3) can be found as follows

$$\begin{aligned}
\frac{\partial U_j^{(k)}}{\partial y_m} &= -\frac{1}{4\mu k_s^2}\left\{k_s^2\delta_{kj}\frac{\partial Y_0\left(k_s r\right)}{\partial r}\ \frac{\partial r}{\partial y_m}\ -\right.\\
&-\left[\frac{\partial^3 Y_0\left(k_p r\right)}{\partial r^3}-\frac{\partial^3 Y_0\left(k_s r\right)}{\partial r^3}\right]\frac{\partial r}{\partial y_m}\ \frac{\partial r}{\partial y_k}\ \frac{\partial r}{\partial y_j}\ -\\
&-\left[\frac{\partial^2 Y_0\left(k_p r\right)}{\partial r^2}-\frac{\partial^2 Y_0\left(k_s r\right)}{\partial r^2}\right]\left[\frac{\partial r}{\partial y_j}\ \frac{\partial^2 r}{\partial y_m\partial y_k}+\frac{\partial r}{\partial y_k}\ \frac{\partial^2 r}{\partial y_m\partial y_j}+\right.\\
&\left.\left.+\frac{\partial r}{\partial y_m}\ \frac{\partial^2 r}{\partial y_k\partial y_j}\right]-\left[\frac{\partial Y_0\left(k_p r\right)}{\partial r}-\frac{\partial Y_0\left(k_s r\right)}{\partial r}\right]\ \frac{\partial^3 r}{\partial y_m\partial y_k\partial y_j}\right\},
\end{aligned} \tag{4.4}$$

where one should take into account that

$$\begin{aligned}
&\frac{dY_0\left(kr\right)}{dr}=-kY_1\left(kr\right)\ ,\qquad \frac{d^2 Y_0\left(kr\right)}{dr^2}=-k^2\left[Y_0\left(kr\right)-\frac{1}{kr}Y_1\left(kr\right)\right],\\
&\frac{d^3 Y_0\left(kr\right)}{dr^3}=-k^2\left[-\frac{1}{r}Y_0\left(kr\right)+\frac{1}{k}\left(\frac{2}{r^2}-k^2\right)Y_1\left(kr\right)\right].
\end{aligned} \tag{4.5}$$

Besides, the components of the divergence and rotor operators applied to these tensors can be directly found in the following form

$$\mathrm{div}\mathbf{U}^{(k)} = \frac{\partial U_1^{(k)}}{\partial y_1} + \frac{\partial U_2^{(k)}}{\partial y_2} = \frac{k_p^3}{4\mu k_s^2} Y_1\left(k_p r\right) \frac{\partial r}{\partial y_k}, \tag{4.6}$$

$$\mathrm{rot}\mathbf{U}^{(1)} = \frac{\partial U_2^{(1)}}{\partial y_1} - \frac{\partial U_1^{(1)}}{\partial y_2} = \frac{1}{4\mu} \frac{\partial Y_0\left(k_s r\right)}{\partial r} \frac{\partial r}{\partial y_2} = -\frac{k_s}{4\mu} Y_1\left(k_s r\right) \frac{\partial r}{\partial y_2}, \tag{4.7a}$$

$$\mathrm{rot}\mathbf{U}^{(2)} = \frac{\partial U_2^{(2)}}{\partial y_1} - \frac{\partial U_1^{(2)}}{\partial y_2} = -\frac{1}{4\mu} \frac{\partial Y_0\left(k_s r\right)}{\partial r} \frac{\partial r}{\partial y_1} = \frac{k_s}{4\mu} Y_1\left(k_s r\right) \frac{\partial r}{\partial y_1}. \tag{4.7b}$$

Here $Y_1(x)$ is the same Neumann function of the order 1.

The following expressions for the partial derivatives of the x-to-y distance are very helpful too, to provide efficient calculations of all quantities involved:

$$\begin{gathered}
\frac{\partial r}{\partial y_i} = \frac{y_i - x_i}{r}, \qquad \frac{\partial^2 r}{\partial y_i^2} = \frac{1}{r}\left[1 - \left(\frac{\partial r}{\partial y_i}\right)^2\right] = \frac{1}{r}\left[1 - \frac{(y_i - x_i)^2}{r^2}\right], \\
\frac{\partial^2 r}{\partial y_m \partial y_i} = -\frac{1}{r}\left(\frac{\partial r}{\partial y_m}\right)\left(\frac{\partial r}{\partial y_i}\right) = -\frac{(y_m - x_m)(y_i - x_i)}{r^3}, \\
\frac{\partial^3 r}{\partial y_i^3} = \frac{3}{r}\frac{\partial r}{\partial y_i}\left[-1 + \left(\frac{\partial r}{\partial y_i}\right)^2\right], \qquad \frac{\partial^3 r}{\partial y_m^2 \partial y_i} = \frac{1}{r^2}\frac{\partial r}{\partial y_i}\left[-1 + 3\left(\frac{\partial r}{\partial y_m}\right)^2\right].
\end{gathered} \tag{4.8}$$

Now let us come back to the fundamental representation (4.1). The components of the stress tensor can be expressed in terms of the displacement tensor in the following way

$$\begin{aligned}
\mathbf{T}_y\left[\mathbf{U}^{(k)}(y,x)\right] &= 2\mu \frac{\partial \mathbf{U}^{(k)}}{\partial n} + \lambda \mathbf{n}\ \mathrm{div}\left(\mathbf{U}^{(k)}\right) + \mu\,\mathrm{rot}\left[\mathbf{n} \times \mathrm{rot}\left(\mathbf{U}^{(k)}\right)\right] = \\
&= \left[2\mu \frac{\partial U_1^{(k)}}{\partial n} + \lambda n_1\ \mathrm{div}\left(\mathbf{U}^{(k)}\right) + \mu\ n_2\left(\frac{\partial U_2^{(k)}}{\partial y_1} - \frac{\partial U_1^{(k)}}{\partial y_2}\right)\right]\mathbf{i} + \\
&\quad + \left[2\mu \frac{\partial U_2^{(k)}}{\partial n} + \lambda n_2\ \mathrm{div}\left(\mathbf{U}^{(k)}\right) - \mu\ n_1\left(\frac{\partial U_2^{(k)}}{\partial y_1} - \frac{\partial U_1^{(k)}}{\partial y_2}\right)\right]\mathbf{j},
\end{aligned} \tag{4.9}$$

where **i** and **j** are unit vectors parallel to the Cartesian coordinate lines x_1 and x_2, respectively.

With the use of Eq.(4.9) representation (4.1) can be rewritten as follows

$$u_k(x) = 2\int_l \left\{ \left[2\mu \frac{\partial U_1^{(k)}}{\partial n} + \lambda n_1 \operatorname{div}\left(\mathbf{U}^{(k)}\right) + \mu n_2 \left(\frac{\partial U_2^{(k)}}{\partial y_1} - \frac{\partial U_1^{(k)}}{\partial y_2} \right) \right] u_1(y) + \right.$$
$$\left. + \left[2\mu \frac{\partial U_2^{(k)}}{\partial n} + \lambda n_2 \operatorname{div}\left(\mathbf{U}^{(k)}\right) - \mu n_1 \left(\frac{\partial U_2^{(k)}}{\partial y_1} - \frac{\partial U_1^{(k)}}{\partial y_2} \right) \right] u_2(y) \right\} dl_y - \tag{4.10}$$
$$-2\int_{l_4} \left\{ U_1^{(k)}(y,x) \quad \sigma(y) + U_2^{(k)}(y,x) \quad \tau(y) \right\} dl_y,$$

which in the more concrete form is equivalent to

$$u_k(x) = 2\int_l \left\{ \left[2\mu \left(\frac{\partial U_1^{(k)}}{\partial y_1} n_1 + \frac{\partial U_1^{(k)}}{\partial y_2} n_2 \right) + \lambda n_1 \left(\frac{\partial U_1^{(k)}}{\partial y_1} + \frac{\partial U_2^{(k)}}{\partial y_2} \right) + \right. \right.$$
$$+ \mu \ n_2 \left(\frac{\partial U_2^{(k)}}{\partial y_1} - \frac{\partial U_1^{(k)}}{\partial y_2} \right) \Bigg] u_1(y) + \left[2\mu \left(\frac{\partial U_2^{(k)}}{\partial y_1} n_1 + \frac{\partial U_2^{(k)}}{\partial y_2} n_2 \right) + \right. \tag{4.11}$$
$$\left. \left. + \lambda n_2 \ \left(\frac{\partial U_1^{(k)}}{\partial y_1} + \frac{\partial U_2^{(k)}}{\partial y_2} \right) - \mu n_1 \left(\frac{\partial U_2^{(k)}}{\partial y_1} - \frac{\partial U_1^{(k)}}{\partial y_2} \right) \right] u_2(y) \right\} dl_y -$$
$$-2\int_{l_4} \left\{ U_1^{(k)}(y,x)\, \sigma(y) + U_2^{(k)}(y,x) \quad \tau(y) \right\} dl_y .$$

More explicit form of these two equations can be achieved if one substitutes here the expressions for the divergence and rotor from Eqs. (4.6), (4.7):

$$u_1(x) = 2\int_l \left\{ \left[2\mu \left(\frac{\partial U_1^{(1)}}{\partial y_1} n_1 + \frac{\partial U_1^{(1)}}{\partial y_2} n_2 \right) + \frac{\lambda n_1 k_p^3}{4\mu k_s^2} Y_1\left(k_p r\right) \frac{\partial r}{\partial y_1} - \right. \right.$$
$$- \frac{n_2 k_s}{4} Y_1(k_s r) \frac{\partial r}{\partial y_2} \Bigg] u_1(y) + \left[2\mu \left(\frac{\partial U_2^{(1)}}{\partial y_1} n_1 + \frac{\partial U_2^{(1)}}{\partial y_2} n_2 \right) + \right. \tag{4.12a}$$
$$\left. \left. + \frac{\lambda n_2 k_p^3}{4\mu k_s^2} Y_1\left(k_p r\right) \frac{\partial r}{\partial y_1} + \frac{n_1 k_s}{4} Y_1(k_s r) \frac{\partial r}{\partial y_2} \right] u_2(y) \right\} dl_y -$$
$$-2\int_{l_4} \left\{ U_1^{(1)}(y,x) \quad \sigma(y) + U_2^{(1)}(y,x) \quad \tau(y) \right\} dl_y,$$

$$u_2(y) = 2\int_l \left\{ \left[2\mu \left(\frac{\partial U_1^{(2)}}{\partial y_1} n_1 + \frac{\partial U_1^{(2)}}{\partial y_2} n_2 \right) + \frac{\lambda n_1 k_p^3}{4\mu k_s^2} Y_1(k_p r) \frac{\partial r}{\partial y_2} + \right. \right.$$
$$\left. + \frac{n_2 k_s}{4} Y_1(k_s r) \frac{\partial r}{\partial y_1} \right] u_1(y) + \left[2\mu \left(\frac{\partial U_2^{(2)}}{\partial y_1} n_1 + \frac{\partial U_2^{(2)}}{\partial y_2} n_2 \right) + \right. \qquad (4.12b)$$
$$\left. \left. + \frac{\lambda n_2 k_p^3}{4\mu k_s^2} Y_1(k_p r) \frac{\partial r}{\partial y_2} - \frac{n_1 k_s}{4} Y_1(k_s r) \frac{\partial r}{\partial y_2} \right] u_2(y) \right\} dl_y -$$
$$-2\int_{l_4} \left\{ U_1^{(2)}(y,x) \quad \sigma(y) + U_2^{(2)}(y,x) \quad \tau(y) \right\} dl_y.$$

Now, when all key formulas have been written out, let us estimate the number of unknown functions and respective number of integral equations. If we consider the set of four different intervals l_1, l_2, l_3, l_4, then over these intervals we have eight unknown Cartesian components of the displacement vector, a couple for each interval, and the pair of contact stresses, functions σ and τ, in the total — ten unknown functions on the four intervals. Eqs. (4.12) applied over each of these four intervals give us eight boundary integral equations. The additional pair of integral equations is given by Eqs. (3.21) since the interval $(-a, a)$ is the same as the interval l_4. We thus have a system of ten boundary integral equations for ten unknown functions.

Application of a standard numerical technique to solve this system of integral equations gives the final solution to the problem.

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ИЗОЛЯЦИЯ ФУНДАМЕНТОВ ОТ СЕЙСМИЧЕСКИХ ВОЛН С ПОМОЩЬЮ ВЯЗКОУПРУГОГО СЛОЯ

© 2007 М.А. Сумбатян[3] В. Дзамполи, М. Ваккаро[4]

В данной работе исследуются гармонические колебания упругого прямоугольника на вязкоупругом слоистом полупространстве. Последнее состоит из упругого полупространства, в которое на некоторой глубине помещен вязкоупругий слой. Комбинируя преобразование Фурье в полупространстве с представлением в рядах в прямоугольнике, удается свести задачу к интегральному уравнению по основанию прямоугольника. Решая данное интегральное уравнение, мы исследуем возможность изоляции основания конструкций в зависимости от вязкоупругих свойств промежуточного слоя, а также от геометрических и физических параметров материалов.

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[3]Сумбатян Межлум Альбертович (sumbat@math.rsu.ru), Механико-математический факультет Ростовского государственного университета, Ростов-на-Дону, 344090, ул. Мильчакова 8а, Россия.

[4]Витторио Дзамполи, Массимо Ваккаро, Университет Салерно, Италия, (vzampoli@unisa.it), (massimo_vaccaro@libero.it).