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DISPERSION APPROACH TO THE PROBLEM OF NEUTRINO MASS-SPLITTING[1]

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The Dashen—Frautschi dispersion method is used to investigate the mass-differences in the neutrino and charge lepton sectors. Here we consider the model in which the charge leptons are described by poles of the $\nu_i\pi^- \to \nu_i\pi^-$ scattering amplitudes where ν_i is the neutrino mass eigenstate with mass m_i The crucial contribution to the mass-splitting arises from the σ-resonance exchanges in the t-channel of these processes. In this way, the relation between mass differences in lepton sectors is derived.

Introduction

Observation [1–5] of solar and atmospheric neutrino fluxes along with the data [6,7] from recent reactor and accelerator experiments have yielded convincing evidence for neutrino oscillations. It is well-known [8,9], neutrino oscillation can occur if, firstly, lepton flavours are not separately conserved, and, secondly, neutrinos are not degenerate in mass. It means that neutrino oscillations cannot be embedded in the framework of the Standard Model because neutrinos are massless particles in this theory. Thus, recent neutrino experiments [1–7] raise the problem of neutrino masses and, in particular, the problem of mass-splitting. The latter seems to be one of the most intriguing subjects of the Particle Physics today.

In this paper, the problem of neutrino mass-splitting is considered in the framework of the Dashen—Frautschi dispersion method [10]. Such an approach looks promising because it is based on the most general principles of the quantum theory (*e.g.*, unitary and analyticity) and, hence, it enables one to go beyond the scope of the Standard Model. In addition to this, it is very impotent to note that many original relations in hadron physics have just obtained by dispersion method. As to Dashen—Frautschi method [10], it has proved to be an extremely helpful tool for calculating mass differences in hadron isomultiplets [11–15].

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The outline of this paper is as follows. In section 2, we propose a simple two-flavour model for calculation of lepton mass-differences and, in accordance with it, formulate dispersion integral relation. In section 3, we calculate the main contributions to the dispersion integral and, as a result of this, obtain the equation connecting mass differences neutrino and charge lepton sectors. In section 4, the results of the calculation are compared with the experimental data on neutrino oscillations. In section 5, we present some concluding remarks.

1. Model

We treat charge leptons as poles of the partial amplitudes in $j^P = \frac{1}{2}^+$ channel for reactions

$$\nu_i + \pi^- \to \nu_i + \pi^-, \tag{1.1}$$

where $\nu_i\,(i = 1, 2, 3)$ is the propagation state of neutrino, *i. e.* the neutrino mass eigenstate with mass $m_i; \pi^-$ is charge pion. This model is based on the assumption that the effect of lepton mass-splitting can be described in term of low-energy parameters.

Besides, we confine the consideration to oscillations between two flavour ν_e and ν_μ that appears to be reasonable taking into account magnitude of the mixing parameter U_{e3} [9]: $|U_{e3}|^2 \leqslant 4 \times 10^{-3}$. In this approximation, the mass eigenstates relate with flavour ones by single mixing angle θ:

$$|\nu_1\rangle = |\nu_e\rangle \cos\theta + |\nu_\mu\rangle \sin\theta, \tag{1.2}$$

$$|\nu_2\rangle = -|\nu_e\rangle \sin\theta + |\nu_\mu\rangle \cos\theta. \tag{1.3}$$

Now adhering to the Dashen—Frautschi procedure [10], we proceed to the investigation of the singularities in the amplitudes for processes (1.1). The amplitudes $A_1(s)$ and $A_2(s)$ have simple poles corresponding to the electron and muon intermediate state in s-channel:

$$\nu_1 + \pi^- \to e^- \to \nu_1 + \pi^-, \quad \nu_2 + \pi^- \to e^- \to \nu_2 + \pi^-, \tag{1.4}$$

$$\nu_1 + \pi^- \to \mu^- \to \nu_1 + \pi^-, \quad \nu_2 + \pi^- \to \mu^- \to \nu_2 + \pi^-. \tag{1.5}$$

We shall start with the analysis of reactions (1.4), as to reactions (1.5), we are concerned with them in Sec. 3.

The electron pole at $s = M_e^2$ dominates the behaviour of the amplitudes in its immediate neighbourhood:

$$A_1(s) \approx R_1^e/(s - M_e^2), \quad A_2(s) \approx R_2^e/(s - M_e^2). \tag{1.6}$$

In this formulas s is the Mandelstam variable, R_1^e and R_2^e are residues in the pole at $s = M_e^2$ where M_e is electron mass. The differences of the residues $\delta R^e = R_2^e - R_1^e$ is related to the singularities of the amplitudes by contour integral [10]:

$$\delta R^e = \frac{1}{\left[D'(M_e^2)\right]^2} \frac{1}{2\pi i} \oint_C \frac{D^2(s)\delta A(s)\, ds}{(s - M_e^2)^2}, \tag{1.7}$$

where $\delta A = A_2(s) - A_1(s)$. There contour C loops clockwise around all the singularities of $D^2(s)\delta A(s)$ in the complex plane s. Formula (1.7) is derived in the framework of the N/D-method [11] where $D(s)$ is the denominator of a partial amplitude. By analogy [10,11], partial amplitude ($j^P = \frac{1}{2}^+$) for processes (1.1) is defined here by the expression

$$A(s) = 1/[(s - M_\pi^2)|\mathbf{k}|]\, e^{i\eta} \sin\eta, \tag{1.8}$$

where η is the phase of the scattering amplitude, $\mathbf{k}$ is the three-momentum of scattering particles in c.m.s., M_π is the π-meson mass. At such a normalization, the amplitude does not contain any kinematical singularities and, along with this, contour integral (1.7) possesses rapid convergence.

2. Mass difference relation

Formula (1.7) is the basic equation for derivation of the mass-differences relation. To obtain it we have to calculate both the left-hand side and the right-hand side of this equation.

First, we find the left-hand side of the equation. By means of formulas (1.2), (1.3) and (1.6) we get the following approximations for $A_1(s)$ and $A_2(s)$ in vicinity of the electron pole:

$$A_1(s) = \langle \nu_1 \pi^- | A | \nu_1 \pi^- \rangle \approx \langle \nu_e \pi^- | A | \nu_e \pi^- \rangle \cos^2\theta \approx \frac{R_e^e \cos^2\theta}{s - M_e^2}, \tag{2.9}$$

$$A_2(s) = \langle \nu_2 \pi^- | A | \nu_2 \pi^- \rangle \approx \langle \nu_e \pi^- | A | \nu_e \pi^- \rangle \sin^2\theta \approx \frac{R_e^e \sin^2\theta}{s - M_e^2}. \tag{2.10}$$

In these formulas R_e^e is the residue of the amplitude $A_e(s)$ in the pole at $s = M_e^2$ which corresponds to the transition

$$\nu_e + \pi^- \to e^- \to \nu_e + \pi^-. \tag{2.11}$$

Standard calculation of the contribution from this process to amplitude (1.8) yields

$$A_e(s) = \frac{f^2 G^2 M_e \cos^2\varphi}{8\pi(s - M_e^2)}, \tag{2.12}$$

where G is the Fermi coupling constant, f is the $\pi \to l + \nu_e$ decay parameter, φ is the Cabibbo angle. Thus, a set of formulas (1.6), (2.9), (2.10) and (2.12) leads to resultant expression for left-hand side of eq. (1.7):

$$\delta R^e = R_2^e - R_1^e = -(1/8\pi) f^2 G^2 M_e \cos^2\varphi \cos 2\theta. \tag{2.13}$$

The second step towards the solution the problem formulated above consists in calculation of contributions to contour integral (1.7) from the singularities of $\delta A(s)$.

However, to begin with we should discuss the question dealing with the form the D-function. According to the N/D-theory [10], D-function has not left singularities, tend to a constant as $s \to \infty$, and $D(M_e^2) = 0$. In the vicinity of the electron pole, the linear approximation for D-function is probably adequate:

$$D(s) = D(M_e^2) + D'(M_e^2)(s - M_e^2) + \cdots \approx s - M_e^2. \qquad (2.14)$$

Here D-function is normalized so that $D'(M_e^2) = 1$.

A detailed analysis of the singularities of $\delta A(s)$ shows that the μ-pole, corresponding to transition (1.5), dominates explicitly among other low-mass singularities. If approximation (2.14) is allowed near μ-pole, then its contribution proves to be equal to

$$\delta R^{\mu} = R_2^{\mu} - R_1^{\mu} = -(1/8\pi) f^2 G^2 M_{\mu} \cos^2\varphi \cos 2\theta. \qquad (2.15)$$

The expression for the residue in the μ-pole is obtained by the same manner as for δR^e in the formula (2.13).

It has already been noted, the μ-exchange in s-channel of (1.5) is a dominant virtual process in low-energy region. Indeed, τ-exchange should be strongly suppressed since the mixing parameter $|U_{e3}|^2 \leqslant 4\times 10^{-3}$ [9]. In addition to this, any one-particle exchange in u-channel is forbidden by lepton flavour conservation law. As to t-channel processes, the contributions from the conventional one-particle intermediate states are canceled in pairs. However, the calculations confined to the consideration of contribution (2.15) only give rise to preposterous result: $M_{\mu} = M_e$. This is readily verified via substitution of expressions (2.13) and (2.15) in left-hand and right-hand sides of eq. (1.7), respectively.

This result is supposed to give evidence to the existence of some non-standard effect being responsible for the lepton mass-splitting. We assume that such an effect might be related with the exchange of a light scalar boson in t-channel provided its interaction with leptons is of the same type as the interaction of the Higgs boson. The σ-resonance is presumably the most natural candidate for this role. At present there is not any explicit interpretation for properties of σ-resonance [16]. Therefore, problem of this particle attract great attention of many researchers [16].

However, here we are interested only in fact that it is a light scalar boson and its basic properties disagree with those of usual mesons [16]. Our additional assumption is that its interaction with ν_1 and ν_2 neutrinos is described by vertex of the following form:

$$V_i = (\sqrt{2}G)^{1/2} m_i. \qquad (2.16)$$

This form is analogous to the one for the interaction of Higgs boson or axion with a massive lepton. Taking all this into consideration, we conjecture that the crucial effect of mass-splitting is caused by the σ-resonance exchanges in t-channel of reactions (1.1):

$$\nu_1 + \bar{\nu}_1 \to \sigma \to \pi^- + \pi^+, \quad \nu_2 + \bar{\nu}_2 \to \sigma \to \pi^- + \pi^+. \qquad (2.17)$$

Due to transitions (2.17), $\delta A(s)$ involves the branch point of a logarithmic type at $|\mathbf{k}(s)|^2 = -\lambda^2/4$ where λ is σ-resonance mass. Going around it in integral (1.7) gives

$$\delta R^{\sigma} = \frac{g_{\sigma\pi\pi}(\sqrt{2}G)^{1/2}}{8\pi\lambda^2}(m_2 - m_1), \qquad (2.18)$$

if equality (2.16) is taken into account and $M_{\pi}^2 \ll \lambda^2$.

Substituting expressions (2.13) and (2.15), (2.18) in the left-hand and right-hand sides of eq. (1.7), respectively, we obtain the following mass-difference relation:

$$\frac{m_2 - m_1}{\cos 2\theta} = \frac{f^2 G^{3/2}\lambda^2 \cos^2\varphi}{2^{1/4} g_{\sigma\pi\pi}}(M_{\mu} - M_e). \qquad (2.19)$$

The same way leads to the analogous relation for the $m_3 - m_2$ and $M_{\tau} - M_{\mu}$ mass-differences.

3. Comparison of results with experimental data

In accordance with analysis [2], the values $m_2^2 - m_1^2 = 5\times10^{-5} eV^2$ and $\tan^2\theta = 0.34$ are the best fit to the data from all solar neutrino experiments. Using these values for oscillation parameters and assuming $m_2^2 \gg m_1^2$, we get $(m_2 - m_1)/\cos 2\theta = 0.014\, eV$. This is the value of the left-hand side of eq. (2.19).

Apart from the σ-resonance parameters, the values of all quantities in right-hand side of the equation are well-known: $M_e = 0.511$ MeV, $M_{\mu} = 106$ MeV, $f \approx 130$ MeV, $G = 1.17 \times 10^{-5}$GeV^{-2} and $\cos\varphi = 0.97$. As far as the σ-resonance mass λ and coupling constant $g_{\sigma\pi\pi}$ are concerned, there appears to be some uncertainty in experimental situation at the moment[16]. Taking this fact into consideration, we find value of the $\lambda^2/g_{\sigma\pi\pi}$ which satisfies eq. (2.19):

$$\lambda^2/g_{\sigma\pi\pi} \approx 250\,\text{MeV}. \qquad (3.20)$$

Certainly, this is only a rough estimate for the parameters. Nevertheless, result (3.20) should be considered as one of prediction following from the model.

Concluding remarks

The principal purpose of this work was to explore the possibility that lepton mass-splitting are induced by low-energy effects. In the framework of the model considered, crucial source of mass-splitting turned out to be connected with the existence of a light scalar boson which is identified as σ-resonance. The mass-differences relation obtained in this way depends essentially on the properties of this boson. Unfortunately, the present experimental data are not enough to verify the validity of eqs. (2.19) and (3.20). Therefore, these relations would be considered as a prediction of the results for forthcoming experiments.

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ДИСПЕРСИОННЫЙ ПОДХОД К ПРОБЛЕМЕ РАСЩЕПЛЕНИЯ МАСС НЕЙТРИНО[3]

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С помощью дисперсионного метода Дашена—Фраучи исследуются разности масс нейтрино и заряженных лептонов. Рассмотрена модель, в которой заряженные лептоны описываются полюсами амплитуд процессов рассеяния $\nu_i\pi^- \to \nu_i\pi^-$, где ν_i — нейтрино с массой m_i.Решающий вклад в расщепление масс вносят σ-резонансные обмены в t-канале этих процессов. Получено соотношение для разностей масс в нейтральном и заряженном лептонных секторах.

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