# CHARGINO-SNEUTRINO AND NEUTRALINO-SMUON CONTRIBUTIONS TO THE ANOMALOUS MUON MAGNETIC MOMENT[1]

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The leading order contribution to the anomalous muon magnetic moment in the minimal supersymmetry is calculated. Symbolic results in the basis of physical fields with arbitrary masses are derived. In the limiting case of large $\tan\beta$ and large degenerate sparticle masses in the loop diagrams the obtained results coincide with the results known from previous publications. However, for different sparticle masses in the loop large deviations from the case of degenerate sparticle masses are obtained. Restrictions for sparticle masses are obtained using the BNL experimental result for the anomalous muon magnetic moment.

## 1. Introduction

The anomalous magnetic moment of the muon (AMM) $a_\mu \equiv (g_\mu - 2)/2$, has been measured with extremely high accuracy (Table 1). The Standard Model electromagnetic,

**Table 1**

The deviation $\delta a_\mu$ of the anomalous magnetic moment $a_\mu^{\rm SM}$ calculated in the Standard Model, from the experimental result $a_\mu^{\rm exp}$ (in units of $10^{-10}$)

| *Quantity* | Value $\times 10^{10}$ | *Reference* | *Comment* |
|---|---|---|---|
| $a_\mu^{\rm exp}$ | 11 659 203(15) | [3, 2] | 1999 data, error 1.3 ppm |
| $a_\mu^{\rm SM}$ | 11 659 176.7(6.7) | [2, 4] | theory (SM), uncertainty 0.6 ppm |
| $\delta a_\mu \equiv a_\mu^{\rm exp} - a_\mu^{\rm SM}$ | 26(16) | [2, 4] | average 1.6$\sigma$ deviation is observed |
| $a_\mu^{\rm MSSM}$ range | [ −6 ; 58 ] | [5] | 2$\sigma$ region |

weak and strong contributions to the AMM are believed to be under control at the sup-ppm level [1] (Table 2). Precise comparison of the Standard Model prediction for AMM with experimental results provides a sensitive critical test of contributions from the new physics [1, 2].

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**Table 2**

The Standard Model contributions to the $a_\mu^{\rm SM}$ (in units of $10^{-10}$)

| *Quantity* | Value $\times 10^{10}$ | *Reference* | *Comment* |
|---|---|---|---|
| $a_\mu^{\rm QED}$ | 11 658 470.6(0.3) | [14] | $\mathcal{O}(\alpha^5)$ estimate |
| $a_\mu^{\rm Had}$ (vacuum polar.) | 684.9(6.4) | [12] | $\mathcal{O}(\alpha^3)$ estimate |
| $a_\mu^{\rm EW}$ | 15.1(0.4) | [15, 16] [17, 18, 19] | 2-loop, $M_H \approx 150$ GeV |
| $a_\mu^{\rm Had}$ (light-by-light) | 8.6(3.2) (average) | [12, 11, 6] [20, 9, 10] | $\mathcal{O}(\alpha^3)$ December, 2001 |

The deviation $\delta a_\mu \equiv a_\mu^{\rm exp} - a_\mu^{\rm SM}$ has changed from $2.6\sigma$ ([1], the E821 result, February 2001) to the average of $1.6\sigma$ ([2, 4], March 2002), which is around 1.7 of the SM electroweak contribution to the $a_\mu$ (Table 2).

The decrease of $\delta a_\mu$ stems mainly from the change in sign ([6], November 2001) of the most important pion pole part in the light-by-light scattering contribution [7, 8] (June 2001). This change of sign has been confirmed in [9, 10, 11] and [12, 13] (December 2001).

Main source of uncertainty in the comparison between theory and experiment is the statistical error. The accuracy of $a_\mu^{\rm exp}$ measurement in the BNL AGS experiment E821 is already of the order of $W^\pm$- and $Z$-boson loop diagram contributions. So the experimental data is precise enough to measure the electroweak SM contribution to the AMM. Furthermore, new experiments are expected to reduce the experimental error of both the $\mu^-$ and $\mu^+$ AMM to the level of $4\times 10^{-10}$ (0.35 ppm) [3, 2] which is better than the present error by a factor of about 4. Their expected error is less than one half of the hadron light-by-light contribution. The major part of theoretical uncertainty in the SM comes from hadronic contributions to the photon vacuum polarization diagram [12, 13, 21] and also from the hadronic component in the light-by-light scattering diagram. But in any case theoretical calculations of the AMM in the SM have the estimated error of less than 1 ppm. At the same time the MSSM radiative corrections to the AMM could be also of the order of 1 ppm in some regions of the MSSM parameter space, providing possibilities to constrain the MSSM parameters using the AMM experimental data.

After the announcement of the E821 result [3] extensive analyses of the MSSM contributions to the AMM were carried out [4]. These analyses showed that the MSSM contribution to the muon anomalous magnetic moment can be large enough in wide regions of the MSSM parameter space [22] defined by the higgsino $\mu$ parameter, the gaugino $M_2$ parameter, and $\tan\beta \equiv v_2/v_1$. The $2\sigma$ level of consistensy with the experimental data $-6 < a_\mu^{\rm MSSM}\times 10^{10} < 58$ is possible if $\mu < 0$ and superparticle masses are large [5]. Main contributions of the MSSM are given by the chargino-sneutrino and neutralino-smuon loop diagrams [1, 23, 24, 25]:

$$a_\mu^{\rm MSSM} = a_\mu(\tilde\chi^\pm) + a_\mu(\tilde\chi^0). \qquad (1)$$

Symbolic expressions for these contributions in the MSSM can be found in [26]. The $a_\mu^{\rm MSSM}$ is enhanced when $\tan\beta$ is large [23]. The result of [26] has been obtained neglecting the gaugino-higgsino mixing, in the limit $\tan\beta >> 1$ and large sparticle masses. If $m^2_{\tilde\mu_L} = m^2_{\tilde\mu_R} = M_2^2 \equiv \tilde m^2$ and the $U(1)_Y$ contribution is neglected [26, 24], the anomalous magnetic moment is given by

$$a_\mu^{\rm MSSM} \approx \frac{5g_2^2}{192\pi^2}\,\frac{m^2}{\tilde m^2}\,{\rm sign}(M_2\mu)\,\tan\beta, \qquad (2)$$

$m$ is the muon mass. The $a_\mu^{\rm MSSM}$ could be further enhanced by the yield of lighter chargino and lighter left-handed slepton, but these contributions are rather small. Dominant contribution to the $a_\mu^{\rm SUSY}$ comes from the chargino-sneutrino loop diagram.

In the physical basis of the chargino and neutralino fields the MSSM contribution to the AMM for the cases $\mu >> M_2$, $\mu << M_2$ and $\mu \approx M_2$ was calculated in [27, 28]. It was shown that the light chargino mixing leads to the increase of the MSSM contribution. The limiting cases for very large and small $\tan\beta$ have been considered in [29], the dependence on $\mu$ and $M_2$ has been analysed in [24].

In this paper we calculate symbolically the AMM in the MSSM with the physical chargino and neutralino fields (chargino and neutralino mass eigenstates) and for arbitrary values of sparticle masses and $\tan\beta$. In Section 2 analytical formulae are presented. In Sections 3 and 4 the limit of large sparticle masses and $\tan\beta$ is discussed. Some useful expressions for the loop integrals can be found in the Appendix.

## 2. Symbolic expressions for $a_\mu$ in the MSSM

We begin with normalizing conventions. The amplitude of muon interaction with the background field reads as

$$i\mathcal{M}(2\pi)\delta(p'^0 - p^0) = -ie\tilde A_\mu^{Cl}(p'-p)\bar u(p')\Gamma^\mu(p',p)u(p), \qquad (3)$$

where $\tilde A_\mu^{Cl}(q)$ is the Fourier transform of the background 4-potential $A_\mu^{Cl}(x)$ and $\Gamma^\mu$ is the one-loop vertex operator

$$\Gamma^\mu = F_1(q^2)\gamma^\mu + F_2(q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2m}\,. \qquad (4)$$

The one-loop correction due to interaction of the muon with the supersymmetric chargino and muon sneutrino (left and right) has the following form:

$$\Lambda^\mu(\tilde\chi^\pm) = \sum_{j=1}^{2}\int\frac{d^4k}{(2\pi)^4}\times$$

$$\times\left\{\frac{(iL_{Lj}P_R + iL_{Rj}P_L)i[\hat p' - \hat k + m_{\tilde\chi_j}]i\gamma^\mu i[\hat p - \hat k + m_{\tilde\chi_j}](iL_{Lj}P_L + iL_{Rj}P_R)}{[(p'-k)^2 - m_{\tilde\chi_j}^2 + i\varepsilon][k^2 - M_{\tilde\nu_{\mu L}}^2 + i\varepsilon][(p-k)^2 - m_{\tilde\chi_j}^2 + i\varepsilon]} + \right.$$

$$\left. + \frac{(iR_{Lj}P_R)i[\hat p' - \hat k + m_{\tilde\chi_j}]i\gamma^\mu i[\hat p - \hat k + m_{\tilde\chi_j}](iR_{Lj}P_L)}{[(p'-k)^2 - m_{\tilde\chi_j}^2 + i\varepsilon][k^2 - M_{\tilde\nu_{\mu R}}^2 + i\varepsilon][(p-k)^2 - m_{\tilde\chi_j}^2 + i\varepsilon]}\right\}. \qquad (5)$$

The Lagrangian of chargino $\tilde\chi_j^+$ $(j = 1, 2)$, muon $\mu$ and muon (left or right) sneutrino $\tilde\nu_{\mu L}$, $\tilde\nu_{\mu R}$, written for chargino mass eigenstates in the basis $\{\tilde\nu_{\mu L}, \tilde\nu_{\mu R}\}$ is obtained as [30, 31, 32]:

$$\mathcal{L}_{\mu\tilde\nu_\mu\tilde\chi^+} = \sum_{j=1}^{2}\{\bar\mu(L_{Lj}P_R + L_{Rj}P_L)\tilde\chi_j^{+C}\tilde\nu_{\mu L} + \bar\mu(R_{Lj}P_R)\tilde\chi_j^{+C}\tilde\nu_{\mu R} +$$

$$+\bar{\tilde{\chi}}_j^{+C}(L_{Lj}P_L + L_{Rj}P_R)\mu\tilde{\nu}^*_{\mu L} + \bar{\tilde{\chi}}_j^{+C}(R_{Lj}P_L)\mu\tilde{\nu}^*_{\mu R}\Big\}, \quad (6)$$

where $L_L$, $R_l$ ($L_R$, $R_R$) are the left (right) chiral amplitudes

$$L_{Lj} \equiv -gV_{j1}, \quad L_{Rj} \equiv Y_\mu U_{j2}, \quad R_{Lj} \equiv Y_{\nu_\mu} U_{j2}, \quad R_{Rj} \equiv 0 \quad (7)$$

with the Yukawa muon $Y_\mu$ and muon neutrino $Y_{\nu_\mu}$ coupling constants

$$Y_\mu = \frac{gm}{\sqrt{2}m_W\cos\beta}, \quad Y_{\nu_\mu} = \frac{gm_{\nu_\mu}}{\sqrt{2}m_W\sin\beta}, \qquad g = \frac{\sqrt{4\pi\alpha}}{\sin\theta_W}, \quad (8)$$

($g$ is the $SU(2)_L$ gauge coupling, $P_L \equiv \frac{1}{2}(\hat{1} - \gamma_5)$ and $P_R \equiv \frac{1}{2}(\hat{1} + \gamma_5)$ are the helicity projectors). $U$ and $V$ are the chargino mixing matrices (see e.g. [32])

$$U_{12} = U_{21} = \frac{\theta_1}{\sqrt{2}}\sqrt{1 + \frac{M_2^2 - \mu^2 - 2m_W^2\cos 2\beta}{W}}, \quad (9)$$

$$U_{22} = -U_{11} = \frac{\theta_2}{\sqrt{2}}\sqrt{1 - \frac{M_2^2 - \mu^2 - 2m_W^2\cos 2\beta}{W}}, \quad (10)$$

$$V_{21} = -V_{12} = \frac{\theta_3}{\sqrt{2}}\sqrt{1 + \frac{M_2^2 - \mu^2 + 2m_W^2\cos 2\beta}{W}}, \quad (11)$$

$$V_{22} = V_{11} = \frac{\theta_4}{\sqrt{2}}\sqrt{1 - \frac{M_2^2 - \mu^2 + 2m_W^2\cos 2\beta}{W}}, \quad (12)$$

wherein

$$W = \sqrt{(M_2^2 + \mu^2 + 2m_W^2)^2 - 4(M_2\mu - m_W^2\sin 2\beta)^2}, \quad (13)$$

$$\{\theta_1, \theta_2, \theta_3, \theta_4\} = \{1, \varepsilon_B, \varepsilon_A, 1\} \qquad \text{for} \quad \tan\beta > 1, \quad (14)$$

$$\varepsilon_A = \mathrm{sign}(M_2\sin\beta + \mu\cos\beta), \quad \varepsilon_B = \mathrm{sign}(M_2\cos\beta + \mu\sin\beta). \quad (15)$$

The chargino masses:

$$m_{\tilde{\chi}_1} = \frac{1}{2}\left|\sqrt{(M_2 - \mu)^2 + 2m_W^2(1 + \sin 2\beta)} - \sqrt{(M_2 + \mu)^2 + 2m_W^2(1 - \sin 2\beta)}\right|, \quad (16)$$

$$m_{\tilde{\chi}_2} = \frac{1}{2}\left\{\sqrt{(M_2 - \mu)^2 + 2m_W^2(1 + \sin 2\beta)} + \sqrt{(M_2 + \mu)^2 + 2m_W^2(1 - \sin 2\beta)}\right\}. \quad (17)$$

The MSSM input parameters are $\tan\beta$, $\mu$, $M_2$. In the following discussion we consider the case $M^2_{\tilde{\nu}_\mu R} \approx M^2_{\tilde{\nu}_\mu L} = M_L^2 + \frac{1}{2}m_Z^2\cos 2\beta \equiv M^2_{\tilde{\nu}_\mu}$, where $M_L$ is the second generation slepton mass parameter, and require the light chagino $\tilde{\chi}_1^+$ is heavier than 100 GeV [15].

The vector part of the expression in (5) can be rewritten in the following form:

$$\Lambda^\mu_{\mathrm{vector}}(\tilde{\chi}^\pm) = -i\sum_{j=1}^{2}\int\frac{d^4k}{(2\pi)^4}\times$$

$$\times\frac{E_{1j}[\hat{p'} - \hat{k} + m_{\tilde{\chi}_j}]\gamma^\mu[\hat{p} - \hat{k} + m_{\tilde{\chi}_j}] + E_{2j}[\hat{p'} - \hat{k} - m_{\tilde{\chi}_j}]\gamma^\mu[\hat{p} - \hat{k} - m_{\tilde{\chi}_j}]}{[(p' - k)^2 - m^2_{\tilde{\chi}_j} + i\varepsilon][k^2 - M^2_{\tilde{\nu}_\mu} + i\varepsilon][(p - k)^2 - m^2_{\tilde{\chi}_j} + i\varepsilon]}, \quad (18)$$

where constants $E_{1j}$ and $E_{2j}$ are

$$E_{1j} = \left(i\frac{L_{Rj}+L_{Lj}}{2}\right)^2 + \left(i\frac{R_{Lj}}{2}\right)^2, \quad E_{2j} = \left(i\frac{L_{Rj}-L_{Lj}}{2}\right)^2 + \left(i\frac{R_{Lj}}{2}\right)^2, \tag{19}$$

or by using equations (7) and (8),

$$E_{1j} = -\frac{\pi\alpha}{\sin\theta_W}\left[\left(\frac{m}{\sqrt{2}m_W\cos\beta}U_{j2} - V_{j1}\right)^2 + \left(\frac{m_{\nu_\mu}}{\sqrt{2}m_W\sin\beta}U_{j2}\right)^2\right], \tag{20}$$

$$E_{2j} = -\frac{\pi\alpha}{\sin\theta_W}\left[\left(\frac{m}{\sqrt{2}m_W\cos\beta}U_{j2} + V_{j1}\right)^2 + \left(\frac{m_{\nu_\mu}}{\sqrt{2}m_W\sin\beta}U_{j2}\right)^2\right]. \tag{21}$$

Separating terms in the numerator of $\Lambda^\mu_{\rm vector}$ in equation (18), which determine the contribution to AMM

$$\bar u(p')\,[\hat p' - \hat k \pm m_{\tilde\chi_j}]\gamma^\mu[\hat p - \hat k \pm m_{\tilde\chi_j}]\,u(p) \Longrightarrow$$

$$\Longrightarrow \bar u(p')\,(\hat k\gamma^\mu\hat k - m\gamma^\mu\hat k - \hat k\gamma^\mu m \mp m_{\tilde\chi_j}\gamma^\mu\hat k \mp \hat k\gamma^\mu m_{\tilde\chi_j})\,u(p) =$$

$$= 2\,\bar u(p')\,(\hat k k^\mu - k^\mu m \mp k^\mu m_{\tilde\chi_j})\,u(p), \tag{22}$$

the expression

$$\Lambda^\mu_{vector}(\tilde\chi^\pm) = -2i\sum_{j=1}^{2}\int\frac{d^4k}{(2\pi)^4}\times$$

$$\times\frac{(E_{1j}+E_{2j})\hat k k^\mu - (E_{1j}+E_{2j})k^\mu m - (E_{1j}-E_{2j})k^\mu m_{\tilde\chi_j}}{[(p'-k)^2 - m^2_{\tilde\chi_j} + i\varepsilon][k^2 - M^2_{\tilde\nu_\mu} + i\varepsilon][(p-k)^2 - m^2_{\tilde\chi_j} + i\varepsilon]}\,. \tag{23}$$

is obtained.

By the aid of Feynman parametrization after the integration over $k$ the magnetic moment operator in the leading order can be found. The factor in front of $(p+p')_\mu/(-$ $-2m)$ determines the chargino-sneutrino contribution to the AMM:

$$a_\mu(\tilde\chi^\pm) = \frac{2}{16\pi^2}\times$$

$$\times\sum_{j=1}^{2}\left\{(E_1+E_2)_j\lambda_2(b_j,a) - \left[(E_1+E_2)_j + (E_1-E_2)_j\frac{m_{\tilde\chi_j}}{m}\right]\lambda_1(b_j,a)\right\}\times$$

$$\times\left(1 - \frac{2\alpha}{\pi}\ln\frac{M_{\tilde\nu_\mu}m_{\tilde\chi_j}}{m^2}\right), \tag{24}$$

where

$$b_j = a + \frac{m^2 - m^2_{\tilde\chi_j}}{2m^2}, \qquad a = \frac{M^2_{\tilde\nu_\mu}}{2m^2}\,. \tag{25}$$

The expression (24) takes account for the leading logarithmic suppression factor

$$1 - \frac{2\alpha}{\pi}\ln\frac{M_{\tilde\nu_\mu}m_{\tilde\chi_j}}{m} \tag{26}$$

which was obtained in [18, 16]. It comes from the leading 2-loop electroweak diagrams. The non-dimensional factors $\lambda_1$, $\lambda_2$ and $\lambda_{\hat 1}$, $\lambda_0$ (in the following they will appear in the neutralino contribution) are defined by integrals contributing to the AMM (see the Appendix). The expression (24) can be rewritten in a different form by substituting there (19):

$$a_\mu(\tilde{\chi}^\pm) = \frac{1}{16\pi^2}\times$$
$$\times \sum_{j=1}^{2} \Big\{ ({L_R}_j^2 + {L_L}_j^2 + {R_L}_j^2)(\lambda_1(b_j,a) - \lambda_2(b_j,a)) + 2{L_R}_j {L_L}_j \frac{m_{\tilde{\chi}_j}}{m} \lambda_1(b_j,a) \Big\} \times$$
$$\times \left(1 - \frac{2\alpha}{\pi} \ln \frac{M_{\tilde{\nu}_\mu} m_{\tilde{\chi}_j}}{m^2}\right). \tag{27}$$

The interaction Lagrangian of neutralino, muon and smuon for the neutralino mass eigenstates $\tilde{\chi}_j^0$ $(j = 1,2,3,4)$ in the basis $\{\tilde{\mu}_L, \tilde{\mu}_R\}$ is [30, 33]:

$$\mathcal{L}_{\mu\tilde{\mu}\tilde{\chi}^0} = \sum_{j=1}^{4} \Big\{ \bar{\mu}(L^0_{Lj} P_R + L^0_{Rj} P_L)\tilde{\chi}^0_j \tilde{\mu}_L + \bar{\mu}(R^0_{Lj} P_R + R^0_{Rj} P_L)\tilde{\chi}^0_j \tilde{\mu}_R +$$
$$+ \bar{\tilde{\chi}}^0_j (L^0_{Lj} P_L + L^0_{Rj} P_R)\mu \tilde{\mu}_L^* + \bar{\tilde{\chi}}^0_j (R^0_{Lj} P_L + R^0_{Rj} P_R)\mu \tilde{\mu}_R^* \Big\}, \tag{28}$$

$$L^0_{Lj} \equiv -g\sqrt{2}\left(\sin\theta_W Z'_{j1} - \frac{\frac{1}{2} + \sin^2\theta_W}{\cos\theta_W} Z'_{j2}\right), \quad L^0_{Rj} = R^0_{Lj} \equiv -g\frac{m}{\sqrt{2}M_W \cos\beta} Z_{j3}, \tag{29}$$

$$R^0_{Rj} \equiv g\sqrt{2}\left(\sin\theta_W Z'_{j1} + \frac{\sin^2\theta_W}{\cos\theta_W} Z'_{j2}\right). \tag{30}$$

$$\Lambda^\mu(\tilde{\chi}^0) = \sum_{j=1}^{4} \int \frac{d^4k}{(2\pi)^4}\times$$
$$\times \Bigg\{ \frac{(iL^0_{Lj} P_R + iL^0_{Rj} P_L) i i [\hat{k} + M_{\tilde{\chi}^0_j}] i (iL^0_{Lj} P_L + iL^0_{Rj} P_R)(p + p' - 2k)^\mu]}{[(p'-k)^2 - m^2_{\tilde{\mu}_L} + i\varepsilon][k^2 - M^2_{\tilde{\chi}^0_j} + i\varepsilon][(p-k)^2 - m^2_{\tilde{\mu}_L} + i\varepsilon]} +$$
$$+ \frac{(iR^0_{Lj} P_R + iR^0_{Rj} P_L) i i [\hat{k} + M_{\tilde{\chi}^0_j}] i (iR^0_{Lj} P_L + iR^0_{Rj} P_R)(p + p' - 2k)^\mu]}{[(p'-k)^2 - m^2_{\tilde{\mu}_R} + i\varepsilon][k^2 - M^2_{\tilde{\chi}^0_j} + i\varepsilon][(p-k)^2 - m^2_{\tilde{\mu}_R} + i\varepsilon]} \Bigg\}. \tag{31}$$

By taking into consideration the GUT relation $M_1 = \frac{5}{3}\tan^2\theta_W M_2$ [30] one can conclude that the four-dimensional neutralino mass matrix depends on the same parameters $\tan\beta$, $\mu$ and $M_2$. The mass matrix can be diagonalized analytically [34] by means of a rotation defined by real matrix. The analytical expressions can be found in [34, 35, 36]. For the neutralino-smuon contribution to the AMM we find

$$a_\mu(\tilde{\chi}^0) = \frac{1}{16\pi^2}\times$$
$$\times \sum_{j=1}^{4} \left\{ 2C_{1j}[\lambda_1(b^0, a^0_j) - \lambda_2(b^0, a^0_j)] + \frac{M_{\tilde{\chi}^0_j}}{m} C_{2j}[\lambda_0(b^0, a^0_j) - 2\lambda_1(b^0, a^0_j)] \right\} \times$$
$$\times \left(1 - \frac{2\alpha}{\pi} \ln \frac{M_{\tilde{\chi}^0_j} m_{\tilde{\mu}}}{m^2}\right), \tag{32}$$

where

$$b^0 = a + \frac{m^2 - m^2_{\tilde{\mu}}}{2m^2}, \qquad a^0_j = \frac{M^2_{\tilde{\chi}^0_j}}{2m^2}. \tag{33}$$

# 3. Large degenerate sparticle masses and $\tan\beta$

For variety of new physics scenarios the leading contribution to the anomalous magnetic moment of a lepton is proportional to $(m_l/m_\Lambda)^2$, where $m_l$ is the mass of a lepton and $m_\Lambda$ is the mass scale of new physics. The anomalous magnetic moment of the muon in the MSSM scenario, see (2), has been obtained in [26] for the case of large degenerate sparticle masses and large $\tan\beta$. Dominant contribution comes from the light chargino-sneutrino loop diagram. This result has been discussed in reviews [1, 13, 37]:

$$a_\mu^{\rm MSSM}(\tilde\chi^\pm) \approx \frac{\alpha}{8\pi\sin^2\theta_W}\frac{m^2}{\tilde m^2}\tan\beta\left(1-\frac{4\alpha}{\pi}\ln\frac{\tilde m}{m}\right)\approx \tag{34}$$

$$\approx ({\rm sign}\mu)\times 140\times 10^{-11}\left(\frac{100\ {\rm GeV}}{\tilde m}\right)^2\tan\beta. \tag{35}$$

Rather small and negative contribution of the neutralino can be accounted by replacing the factor 140 in the last expression by the factor 130 [1].

The result (34) can be reproduced if we keep only the term proportional to $L_{Rj}L_{Lj}\frac{m_{\tilde\chi_j}}{m}$ in the exact expression (27) omitting there at the same time the logarithmic suppression factor [4]

$$a_\mu^{\rm MSSM}(\tilde\chi^\pm) \approx \frac{1}{8\pi^2}\sum_{j=1}^{2}L_{Rj}L_{Lj}\frac{m_{\tilde\chi_j}}{m}\lambda_1(b_j,a) \tag{36}$$

Only the terms linear in $m_{\tilde\chi_j}$ in the numerator of (5) contribute to the approximate formula (36). If for $\lambda_1$ we use the approximation (52) of large unequal masses in the loop diagram (see the Appendix), then our result coincides with the corresponding expression in [26].

For the case of degenerate sparticle masses we substitute (55) instead of $\lambda_1$ and use the approximation $L_{R1}L_{L1}\approx L_{R2}L_{L2}\equiv L_RL_L$

$$a_\mu^{\rm MSSM}(\tilde\chi^\pm) \approx \frac{1}{12\pi^2}\frac{m}{\tilde m}L_RL_L\left(1-\frac{4\alpha}{\pi}\ln\frac{\tilde m}{m}\right), \tag{37}$$

easily reproducing (using (7) and (8)) the formula (34). [5]

# 4. Large unequal sparticle masses and $\tan\beta$

For the comparison of one-loop integrals in the case of large unequal superparticle masses with the case of large equal masses it is convenient to introduce the functions $l_0(z)$, $l_1(z)$ and $l_2(z)$ (see the Appendix for details),

$$l_0(z)\equiv\frac{\lambda_0(v,z)}{v}=\frac{2}{z-1}-\frac{2}{(z-1)^2}\ln z\,,\qquad z\equiv\frac{m_1^2}{M^2}\,, \tag{38}$$

$$l_1(z)\equiv\frac{\lambda_1(v,z)}{v/3}=3\left(\frac{1}{2(z-1)}-\frac{1}{(z-1)^2}+\frac{1}{(z-1)^3}\ln z\right)\,, \tag{39}$$

$$l_2(z)\equiv\frac{\lambda_2(v,z))}{v/4}=4\left(\frac{1}{3(z-1)}-\frac{1}{2(z-1)^2}+\frac{1}{(z-1)^3}-\frac{1}{(z-1)^4}\ln z\right)\,. \tag{40}$$

[4] The product $L_{Rj}L_{Lj}$ is denoted by $C^LC^R$ in [26]. These factors are contained in the terms which have a helicity flip of the internal fermion line.

[5] In the expression (2) the logarithmic suppression factor is omitted.

The $l_0(z)$, $l_1(z)$, $l_2(z)$ functions are plotted in Fig. 1 and 2, see also Tables 3 and 4.

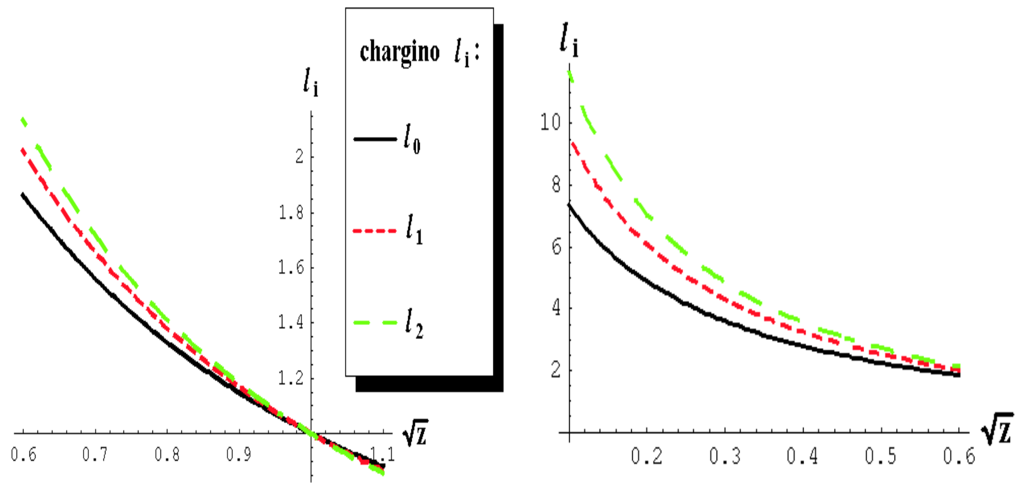


Fig. 1. Integrals for the chargino type contribution. The left plot of $l_i(z = \frac{m^2_{\tilde{\chi}_1}}{M^2_{\tilde{\nu}_\mu}})$ for $0.6 < \frac{m_{\tilde{\chi}_1}}{M_{\tilde{\nu}_\mu}} < 1.1$, the right plot of $l_i(z)$ for $0.1 < \frac{m_{\tilde{\chi}_1}}{M_{\tilde{\nu}_\mu}} < 0.6$

**Table 3**

Comparison between $\lambda_i(v, z)$ and $\lambda_i(v)$ for the first chargino case

| $m_1/M \equiv m_{\tilde{\chi}_1}/M_{\tilde{\nu}_\mu}$ | 1/10 | 1/5 | 1/4 | 1/3 | 1/2 | 1 |
|---|---|---|---|---|---|---|
| $l_0(z)$ | 7.38 | 4.90 | 4.18 | 3.31 | 2.26 | 1 |
| $l_1(z)$ | 9.66 | 6.10 | 5.08 | 3.90 | 2.52 | 1 |
| $l_2(z)$ | 11.67 | 7.08 | 5.80 | 4.35 | 2.71 | 1 |

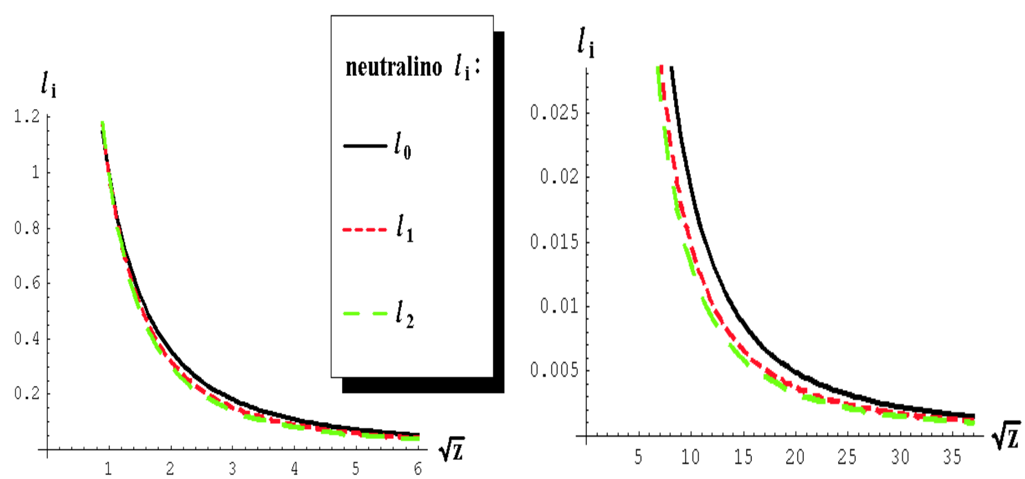


Fig. 2. Integrals for the neutralino (z=$m^2_{\tilde{\mu}}/M^2_{\tilde{\chi}^0_j}$) or heavy chargino type contribution

**Table 4**

Comparison between $\lambda_i(v,z)$ and $\lambda_i(v)$ for the neutralino case

| $m_{\tilde{\mu}}/M_{\tilde{\chi}^0_j}$ | 2 | 3 | 4 | 5 | 10 | 20 |
|---|---|---|---|---|---|---|
| $l_0(z)$ | 0.36 | 0.18 | 0.11 | 0.07 | 0.02 | 0.005 |
| $l_1(z)$ | 0.32 | 0.15 | 0.09 | 0.06 | 0.01 | 0.004 |
| $l_2(z)$ | 0.30 | 0.14 | 0.08 | 0.05 | 0.01 | 0.003 |

In the important case of large unequal sparticle masses the AMM contains different contributions of the two charginos

$$a_\mu^{\rm MSSM}(\tilde{\chi}_1^\pm,\tilde{\chi}_2^\pm) \approx \frac{l_1(z_1)+l_1(z_2)}{2}\cdot({\rm sign}\mu)\times 14\times 10^{-10}\left(\frac{100\ {\rm GeV}}{M_{\tilde{\nu}_\mu}}\right)^2 \tan\beta. \qquad (41)$$

For the $1.6\sigma$ deviation of the $\delta a_\mu = 26\pm 16$ (see Table 1) the sneutrino mass variation range is approximated by

$$M_{\tilde{\nu}_\mu}^{\rm min,max} \longleftrightarrow \sqrt{\frac{l_1(z_1)+l_1(z_2)}{2}}\cdot\left(\frac{100\ {\rm GeV}}{\sqrt{1.9\pm 1.1}}\right)\sqrt{\tan\beta}. \qquad (42)$$

## 5. Conclusion

The leading order contribution (41) to the anomalous muon magnetic moment in the basis of physical chargino and neutralino fields with arbitrary masses has been calculated. The dependence of the AMM limits on the masses of light chargino and muon sneutrino is shown in Fig. 3a, 3b for $\tan\beta$ =25 and $\tan\beta$ =40. For the case under consideration the parameters $\mu$ and $M_2$ are large (of the order of TeV), so the contribution of heavy chargino is very small and can be neglected. In the narrow area between the thin shaded lines the MSSM contribution to the AMM provided by the light chargino is equal to the electroweak contribution of the Standard Model $(15.1\pm0.4)\times10^{-10}$. In the region of $m_{\tilde{\chi}_1}, m_{\tilde{\chi}_2}$ plane marked by the thick dashed lines the deviation of the AMM from the Standard Model value is less than $1.6\sigma$. Thus at a given large enough $\tan\beta$ the existing experimental data on the AMM favor the values of the light chargino mass in the ranges around 100 GeV or around 500 GeV at the $m_{\nu_\mu}$ ~500 GeV and then decreasing when the muon sneutrino mass increases, which is different from the approximation of degenerate masses shown in Fig. 3 by the solid line. If the sparticle mass difference is sufficiently large (more than 100 GeV), the MSSM contribution to the AMM is substantially different from the approximation of degenerate sparticle masses.

The similar situation is observed if we plot the AMM limits in the variables $m_{\tilde{\chi}_1}, m_{\tilde{\chi}_2}$ and $\mu, M_2$ of the MSSM parameter space. The dependence of the AMM limits on the masses of two charginos $m_{\tilde{\chi}_1}, m_{\tilde{\chi}_2}$ is shown in Fig. 4a, 4b for $\tan\beta$ =25 and $\tan\beta$ =40. The muon sneutrino mass is fixed at the value $M^2_{\nu_\mu} = M_L^2 + m_Z^2\cos2\beta/2$ ($M_L$ is the slepton mass). In the narrow area between the thin dashed lines the MSSM contribution to the AMM coming from the light chargino and neutralino is equal to the electroweak contribution of the Standard Model. The area marked by the thick dashed lines is the region of the $m_{\tilde{\chi}_1}, m_{\tilde{\chi}_2}$ plane where the deviation of the AMM from the Standard Model value is less than $1.6\sigma$. The same denominations are used in Fig. 5a, 5b for the AMM limits in the $\mu$, $M_2$ parameter plane.

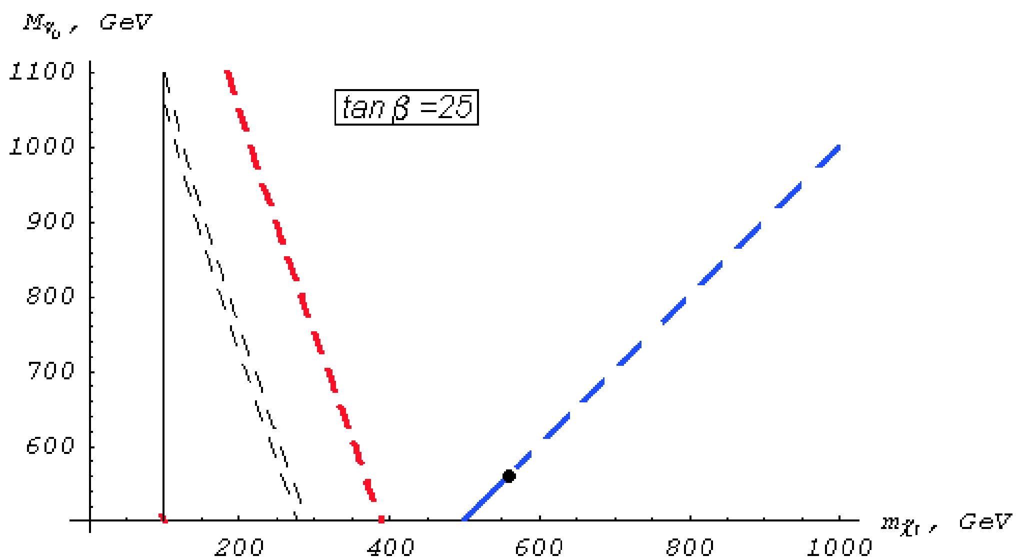


Fig. 3a. The dependence of AMM $a_\mu(\tilde\chi_1^\pm)$ on the masses of light chargino and muon sneutrino, tan$\beta = 25$. In the area between the thin dashed lines $a_\mu^{SUSY}(\chi_1^\pm) = (15.1 \pm 0.4) \times 10^{-10}$. In the region of the $m_{\tilde\chi_1}, m_{\tilde\nu_\mu}$ plane to the left from the thick dashed line $a_\mu^{SUSY}(\chi_1^\pm) = (26 \pm 16) \times$ $\times 10^{-10}$. Solid line corresponds to the degenerate chargino-sneutrino masses

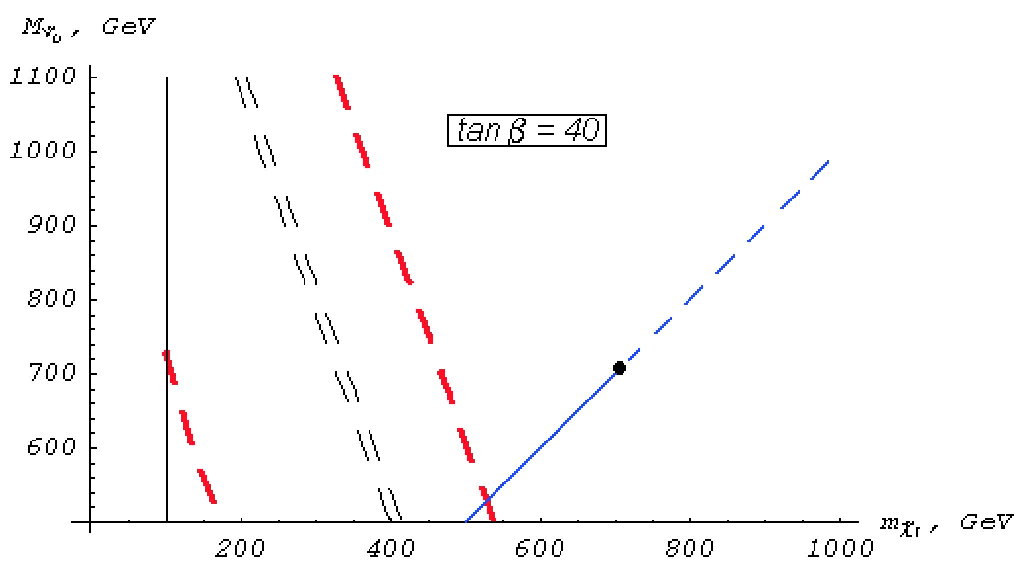


Fig. 3b. The dependence of AMM $a_\mu(\tilde\chi_1^\pm)$ on the masses of light chargino and muon sneutrino, tan$\beta = 40$. In the area between the thin dashed lines $a_\mu^{SUSY}(\chi_1^\pm) = (15.1 \pm 0.4) \times 10^{-10}$. In the region of the $m_{\tilde\chi_1}, m_{\tilde\nu_\mu}$ plane between the thick dashed lines $a_\mu^{SUSY}(\chi_1^\pm) = (26 \pm 16) \times 10^{-10}$. Solid line corresponds to the degenerate chargino-sneutrino masses

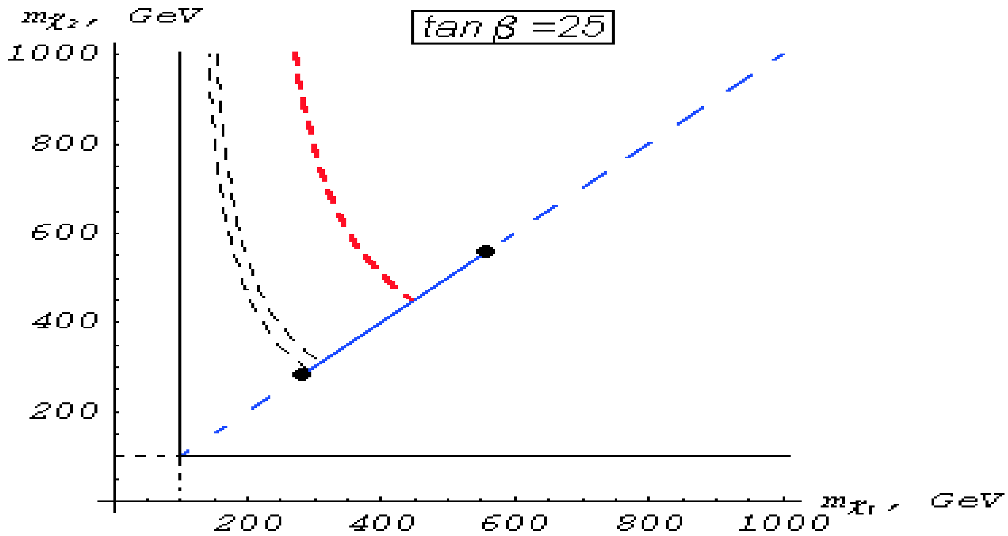


Fig. 4a. The $a_\mu(\tilde{\chi}_1^\pm, \tilde{\chi}_2^\pm)$ dependence on chargino masses for $\tan\beta = 25$

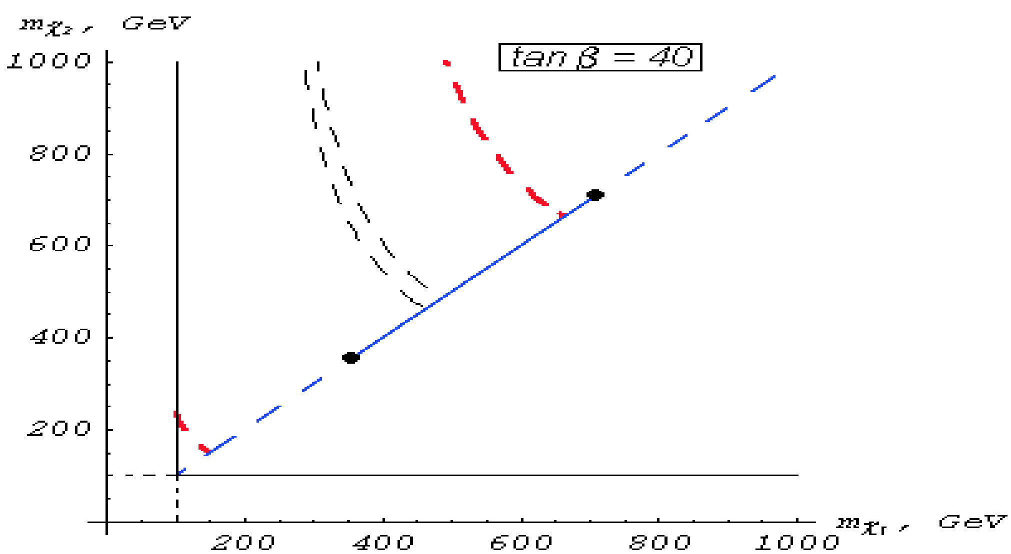


Fig. 4b. The $a_\mu(\tilde{\chi}_1^\pm, \tilde{\chi}_2^\pm)$ dependence on chargino masses for $\tan\beta = 40$

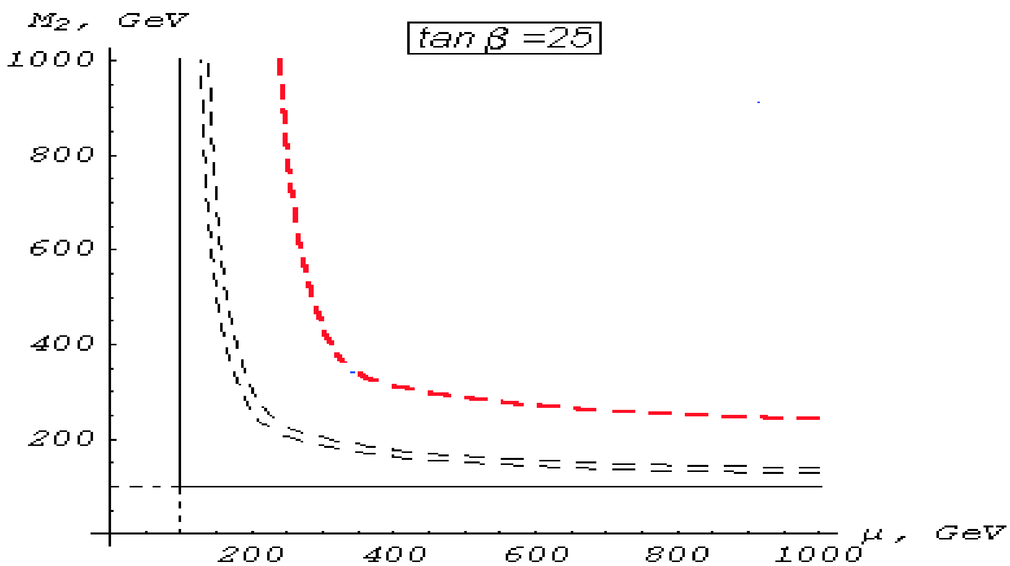


Fig. 5a. The $a_\mu(\tilde{\chi}_1^\pm, \tilde{\chi}_2^\pm)$ dependence on the $\mu, M_2$ parameters, $\tan\beta = 25$

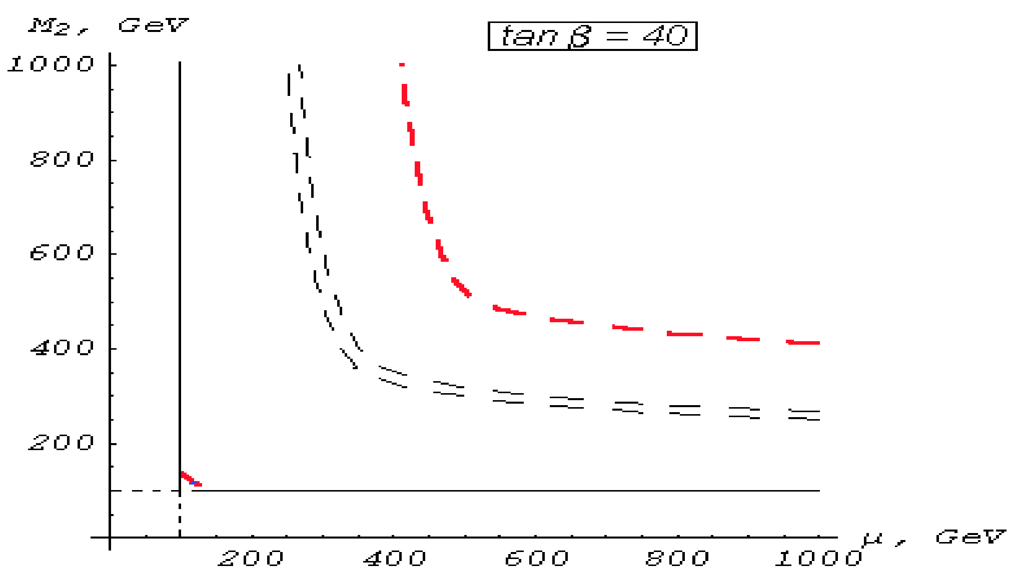


Fig. 5b. The $a_\mu(\tilde{\chi}_1^\pm, \tilde{\chi}_2^\pm)$ dependence on the $\mu, M_2$ parameters, $\tan\beta = 40$

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## Appendix. One-loop integrals contributing to the AMM

One-loop contribution to the anomalous magnetic moment of the charged lepton $a_l$, may be presented as following decomposition

$$a_l = \frac{i}{16\pi^2} \cdot \sum C_{\{0;1;\hat{1};2\}} \cdot \lambda_{\{0;1;\hat{1};2\}}(m^2/m_1^2, m^2/M^2). \tag{43}$$

Scalar dimensionless quantities $\lambda_{\{0;1;\hat{1};2\}}$ depending on relations between loop and external masses are determined by oneloop integrals of four types:

$$\int \frac{d^4k}{(2\pi)^4} \frac{\bar{u}(p')\,\{\, m(p+p')^\mu\,;\, mk^\mu\,;\, (p+p')^\mu \hat{k}\,;\, \hat{k}k^\mu \,\}\, u(p)}{[(p'-k)^2 - m_1^2 + i\varepsilon][k^2 - M^2 + i\varepsilon][(p-k)^2 - m_1^2 + i\varepsilon]} \Longrightarrow \tag{44}$$

$$\Longrightarrow \frac{i}{16\pi^2} \cdot \lambda_{\{0;1;\hat{1};2\}}(m^2/m_1^2, m^2/M^2) \cdot \left[ \bar{u}(p') \left( \frac{i\sigma^{\mu\nu} q_\nu}{2m} \right) u(p) \right],$$

where symbol "$\Longrightarrow$" means we consider contributions to the anomalous magnetic moment only.

The integration in (44) leads to the following analytical results for common case:

$$\lambda_{\{0;1;\hat{1};2\}}(m^2/m_1^2, m^2/M^2) = \int\limits_0^1 dt \frac{\{\, 2t; t^2; 2t^2; t^3 \,\}}{t^2 - 2bt + 2a - i\varepsilon}\,, \tag{45}$$

then

$$\lambda_0(b,a) = \ln(z) + 2bL(b,a), \tag{46}$$

$$\lambda_1(b,a) = 1 + b\ln(z) + 2(b^2 - a)L(b,a), \qquad \lambda_{\hat{1}} = 2\lambda_1, \tag{47}$$

$$\lambda_2(b,a) = \frac{1}{2} + 2b + (2b^2 - a)\ln(z) + 2(2b^3 - 3ab)L(b,a), \tag{48}$$

where $L(b,a)$ is the continuous function $(a > 0)$

$$L(b,a) = \begin{cases} \dfrac{1}{2K(b,a)} \ln \left| \dfrac{-b + 2a + K(b,a)}{-b + 2a - K(b,a)} \right|, & b^2 > 2a, \qquad K(b,a) = \sqrt{|b^2 - 2a|}, \\ \dfrac{1}{2a - b}\,, & b^2 = 2a, \\ \dfrac{1}{K(b,a)} \left( \arctan \dfrac{1-b}{K(b,a)} + \arctan \dfrac{b}{K(b,a)} \right), & b^2 < 2a, \end{cases} \tag{49}$$

$$b \equiv \frac{1 - z + v}{2v}\,, \quad a \equiv \frac{1}{2v}\,, \qquad v \equiv \frac{m^2}{M^2}\,, \quad z \equiv \frac{m_1^2}{M^2}\,. \tag{50}$$

## A.1. Large ($m_1, M \gg m$) unequal masses in the loop

$$\lambda_0(v,z) = \left( \frac{2}{z-1} - \frac{2}{(z-1)^2} \ln z \right) v = = \frac{2m^2}{M^2} \cdot \frac{-1}{(1-z)^2} (1 - z + \ln z) \,, \tag{51}$$

$$\lambda_1(v,z) = \left( \frac{1}{2(z-1)} - \frac{1}{(z-1)^2} + \frac{1}{(z-1)^3} \ln z \right) v = = \frac{3m^2}{2M^2} \cdot \frac{-1}{(1-z)^3} \left( 1 - \frac{4}{3} z + \frac{1}{3} z^2 + \frac{2}{3} \ln z \right) \,, \tag{52}$$

$$\lambda_2(v,z) = \left( \frac{1}{3(z-1)} - \frac{1}{2(z-1)^2} + \frac{1}{(z-1)^3} - \frac{1}{(z-1)^4} \ln z \right) v = = \frac{11m^2}{6M^2} \cdot \frac{-1}{(1-z)^4} \left( 1 - \frac{18}{11} z + \frac{9}{11} z^2 - \frac{2}{11} z^3 + \frac{6}{11} \ln z \right) . \tag{53}$$

## A.2. Large equal $m_1 = M$ ($z = 1$) masses in the loop

$$\lambda_0(v) = v = \frac{m^2}{M^2} \,, \tag{54}$$

$$\lambda_1(v) = \frac{v}{3} = \frac{m^2}{3M^2} \,, \tag{55}$$

$$\lambda_2(v) = \frac{v}{4} = \frac{m^2}{4M^2} \,. \tag{56}$$

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